Math 1431 DAY 15
Dr. Melahat Almus
almus@math.uh.edu

OFFICE HOURS: MWF 11-11:30am, MW 1-2:15pm at 621 PGH

If you e-mail me, please mention your course (1431) in the subject line.

Check your CASA account for quiz due dates; don’t miss any quizzes.

BUBBLE IN PS ID VERY CAREFULLY! If you make a bubbling mistake, your scantron will not be saved in the system and you will not get credit for it even if you turned it in. Bubble in Popper Number.

Be considerate of others in class. Respect your friends and do not distract anyone during the lecture.

Popper #

\[ x(t) = -16 t^2 + 320 t. \]

**Question#** How long does it take to reach the maximum height?

a) 10 seconds  
b) 5 seconds  
c) 15 seconds  
d) 20 seconds  
e) None

**Question#** What is the maximum height?

a) 1600  
b) 3200  
c) 3040  
d) 2400  
e) None
**Question#** What is the total time of travel?

a) 10 seconds  
   b) 20 seconds  
   c) 16 seconds  
   d) 32 seconds  
   e) None

**Question#** What is the speed on impact?

a) -320 ft/sec  
   b) 320 ft/sec  
   c) 160 ft/sec  
   d) -160 ft/sec  
   e) None
Section 3.2 Mean Value Theorem

Theorem: Rolle’s Theorem

Suppose that $f$ is continuous on the closed interval $[a,b]$ and differentiable on the open interval $(a,b)$.

If $f(a) = f(b)$, then there is at least one number $c$ in $(a,b)$ for which $f'(c) = 0$.

The essence of Rolle’s theorem may be seen on these pictures:

Rolle’s theorem is sometimes stated as follows:

Suppose that $f$ is continuous on the closed interval $[a,b]$ and differentiable on the open interval $(a,b)$. If $f(a) = f(b) = 0$, then there is at least one number $c$ in $(a,b)$ for which $f'(c) = 0$.

That is, Rolle’s theorem tells us that between any two roots of $f$, there must be a root of $f'$. 
**Example:** Verify that the Rolle’s Theorem applies to the function

\[ f(x) = x^2 - 4x + 2, \quad [0, 4] \]

Find all points in this interval that satisfy Rolle’s Theorem.

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**Example:** Verify that the Rolle’s Theorem applies to the function

\[ f(x) = \cos(2x), \quad [0, \pi] \]

Find all points in this interval that satisfy Rolle’s Theorem.
The mean-value theorem is a generalization of the Rolle’s Theorem.

**Theorem: The Mean-Value Theorem**

Suppose that $f$ is continuous on the closed interval $[a,b]$ and differentiable on the open interval $(a,b)$. There is at least one number $c$ in $(a,b)$ for which

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Of course, there may be more than one point where the tangent line is parallel to the secant line.
**Example:** Verify that the Mean Value Theorem applies to the function

\[ f(x) = x^3 - 2x + 1, \quad [0,2] \]

Find all points in this interval that satisfy Mean Value Theorem.

**Example:** Verify that the Mean Value Theorem applies to the function

\[ f(x) = x + \frac{1}{x}, \quad [1,4] \]

Find all points in this interval that satisfy Mean Value Theorem.
Example: Verify that the Mean Value Theorem applies to the function

\[ f(x) = \frac{x + 2}{x}, \quad [-1, 2] \]

Find all points in this interval that satisfy Mean Value Theorem.

Example: Verify that the Mean Value Theorem applies to the function

\[ f(x) = x^{2/3}, \quad [-1, 1] \]

Find all points in this interval that satisfy Mean Value Theorem.
**Example:** At how many points between 0 and 10 does the function satisfy the Mean Value theorem?
Exercise: Does Rolle’s Theorem apply to the given function over the indicated interval? Does Mean Value Theorem apply? If a theorem applies, find the values of c in the given interval that satisfy the theorem.

<table>
<thead>
<tr>
<th>Function</th>
<th>Rolle’s Theorem?</th>
<th>MVT?</th>
<th>Value(s) of c</th>
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<tbody>
<tr>
<td>$f(x) = x^3 - 12x$, $[0,3]$</td>
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<tr>
<td>$f(x) = \cos(2x)$, $[0,\pi]$</td>
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<tr>
<td>$f(x) = 2\sqrt{x} - x$, $[0,4]$</td>
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<td>$f(x) = 2\sqrt{x} - x$, $[0,1]$</td>
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<td>$f(x) = 2x^{1/3}$, $[-1,1]$</td>
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<td>$f(x) = 2x^{1/3}$, $[1,8]$</td>
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<td>$f(x) = \frac{x^2}{x-2}$, $[1,3]$</td>
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<td>$f(x) = \frac{x^2}{x-2}$, $[0,1]$</td>
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Exercise: Verify that the Mean Value Theorem applies to the function

$$f(x) = \sin(4x), \quad \left[0, \frac{\pi}{2}\right]$$

Find all points in this interval that satisfy Mean Value Theorem.
Section 3.3 – Intervals of Increase and Decrease and Extreme Values

**Definition:** Let $f$ be a function whose domain includes an interval $I$.

We say that $f$ is **increasing** on $I$ if for every two numbers $x_1$, $x_2$ in $I$,

$$x_1 < x_2 \implies f(x_1) < f(x_2).$$

We say that $f$ is **decreasing** on $I$ if for every two numbers $x_1$, $x_2$ in $I$,

$$x_1 < x_2 \implies f(x_1) > f(x_2).$$

If the graph of a function is given, it is very easy to find the intervals of increase and decrease. Simply observe whether the $y$–values are going up or down.
Example: The graph of \( f(x) \) is given below:

The function is increasing over the intervals:

The function \( f \) is decreasing over the intervals:
The function \( f \) is increasing over the intervals:

\( f \) is decreasing over the intervals:
What if the function is given by a formula?

\[ f(x) = x^2 - 1 \]

\[ f(x) = \sin x \]

\[ f(x) = x^4 - 5x^3 + x - 2 \quad \text{or} \quad f(x) = x^2 \cos x? \]

**POPPER #**

\[ f(x) = x^2 + 1, \ [ -1,1 ] \]

**Question#**  Does Rolle’s Theorem apply?  

a) Yes  b) No

**Question#**  Does Mean Value Theorem apply?  

a) Yes  b) No

\[ f(x) = \frac{x^2 + 1}{x}, \ [ -1,1 ] \]

Does Mean Value Theorem apply?  

a) Yes  b) No

**Question#**  

\[ f(x) = \frac{x^2 + 1}{x}, \ [ 1,2 ] \]

Does Mean Value Theorem apply?  

a) Yes  b) No