Math 1431 DAY 20
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OFFICE HOURS: MWF 11-11:30, 1-2:15pm, 610 PGH.
If you e-mail me, please mention your course (1431) in the subject line.
Check your CASA account for quiz due dates; don’t miss any quizzes.

BUBBLE IN PS ID VERY CAREFULLY! If you make a bubbling mistake, your scantron will not be saved in the system and you will not get credit for it even if you turned it in. Bubble in Popper Number.

Be considerate of others in class. Respect your friends and do not distract anyone during the lecture.

Popper#

Question #

What is/are the point(s) of inflection for $f(x) = 3x^5 - 10x^4 + 1$?

A. $x=0$ and $x=2$
B. $x=0$ and $x=1$
C. $x=1$ and $x=2$
D. $x=2$
E. $x=-2$
**Question#**

The graph of \( f'(x) \) (first derivative!) is given below.

When is the function \( f(x) \) decreasing?

A. \((-\infty, 0)\) and \((2, \infty)\)

B. \((-\infty, 1)\)

C. \((0,2)\)

D. \((1, \infty)\)

E. \((-\infty, \infty)\)
**Question#**

The graph of $f'(x)$ (first derivative!) is given below. When is the function $f(x)$ concave up?

A. $(-\infty, 0)$ and $(2, \infty)$

B. $(-\infty, 1)$

C. $(0, 2)$

D. $(1, \infty)$

E. $(-\infty, \infty)$

**Question#**

The graph of $f'(x)$ is shown below. Give the number of local minimums for $f(x)$.

- a. 0
- b. 1
- c. 2
- d. 3
- e. 4

**Question#**

The graph of $f'(x)$ is shown below. Give the number of points of inflection of $f(x)$.

- a. 3
- b. 4
- c. 5
- d. 6
- e. 7
Section 3.6 – Curve Sketching

Note: Make sure you know how to find Vertical and Horizontal Asymptotes!

Vertical Tangents

Suppose that \( f(x) \) is continuous at \( x = c \).

If \( f'(x) \to \infty \) or \( f'(x) \to -\infty \) as \( x \to c \), then we say that the function has a **vertical tangent** at the point \((c, f(c))\).

Example: \( f(x) = (x - 9)^{1/3} \)
**Vertical Cusps**

Suppose that $f(x)$ is continuous at $x = c$.

If $f'(x) \to \infty$ as $x \to c$ from one side and $f'(x) \to -\infty$ as $x \to c$ from the other side, then we say that the function has a **vertical cusp** at the point $(c, f(c))$.

**Example:** $f(x) = (x - 4)^{2/7}$
Using Calculus to graph a function.

1. Determine the **domain** of the function \( f \). Find any vertical asymptotes and study the behavior of \( f \) as \( x \to \pm \infty \).

2. Determine any **intercepts** of the function. To find the \( x \)–intercepts, we need to solve the equation \( f(x) = 0 \) and to find the \( y \)–intercepts, evaluate the function at 0 (if 0 is in the domain of \( f \)).

3. Find the **first derivative**, \( f' \). Determine any critical points, intervals of increase/decrease, local extreme points, vertical tangents and cusps.

4. Find the **second derivative**, \( f'' \). Study the sign of \( f'' \) to understand concavity of the function and determine any points of inflection.

5. Plot the **points of interest** (intercepts, local or absolute extreme points, points of inflection).

6. Sketch the graph of \( f \) using the information gathered in the previous steps. Make sure that the function has the right shape (concaves up/down, rises/falls) on the corresponding intervals.

### Summary of Graphical Features

<table>
<thead>
<tr>
<th>Condition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f'(x) &gt; 0 ) / ( f'(x) &lt; 0 )</td>
<td>decreasing / increasing</td>
</tr>
<tr>
<td>( f''(x) &gt; 0 ) / ( f''(x) &lt; 0 )</td>
<td>concave up / down</td>
</tr>
<tr>
<td>( \lim_{x \to \pm \infty} f(x) = b ) or ( \lim_{x \to \pm \infty} f(x) = b )</td>
<td>horizontal asymptote at ( y = b )</td>
</tr>
<tr>
<td>( \lim_{x \to a} f(x) \to \pm \infty ) or ( \lim_{x \to a} f(x) \to \pm \infty )</td>
<td>vertical asymptote at ( x = a )</td>
</tr>
<tr>
<td>( \lim_{x \to a} f'(x) \to \pm \infty )</td>
<td>vertical tangent at ( x = a )</td>
</tr>
<tr>
<td>( \lim_{x \to a} f''(x) \to \pm \infty )</td>
<td>cusp at ( x = a )</td>
</tr>
<tr>
<td>( \lim_{x \to a^+} f'(x) \to \pm \infty )</td>
<td>“corner” at ( x = a )</td>
</tr>
</tbody>
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Graphing a Radical Function

Example: Graph $f(x) = (2 - x)^{\frac{4}{5}}$
EXERCISE: Graph $f(x) = x(x-1)^{\frac{1}{3}}$

$f'(x) = \frac{4x - 3}{2} \cdot \frac{2}{3(x-1)^{\frac{2}{3}}}$

$f''(x) = \frac{4x - 6}{5} \cdot \frac{5}{9(x-1)^{\frac{5}{3}}}$
Graphing a Polynomial

Example: Find the domain, critical points, relative extrema, intervals of increase decrease, concavity, and graph $f(x) = -4x^3 - 6x^2 + 24x + 12$
Graphing a Rational Function

Example: Graph \( f(x) = \frac{2x}{x^2 + 1} \).

Domain:

Intercepts:

Asymptotes:

\[
f'(x) = \frac{-2x^2 + 2}{(x^2 + 1)^2}
\]

\[
f''(x) = \frac{4x^3 - 12x}{(x^2 + 1)^3}
\]
POPPER #

Question# Which function has a cusp at \( x = 1 \)?

a) \( f(x) = x^{1/3} \)

b) \( f(x) = (x - 1)^{3/5} \)

c) \( f(x) = (x - 1)^{2/5} \)

d) \( f(x) = (x + 1)^{3/5} \)

e) None

Question# Which function has a vertical tangent at \( x = 3 \)?

a) \( f(x) = 3 - x^{1/3} \)

b) \( f(x) = (x - 3)^{3/5} \)

c) \( f(x) = (x - 3)^{2/5} \)

d) \( f(x) = (x + 3)^{4/5} \)

e) None
Match the function with its first derivative.

Functions:

A.  

B.  

C.  

D.  

E.  

[Graphs of different functions with corresponding derivatives]