Math 1431
Dr. Melahat Almus
almus@math.uh.edu

OFFICE HOURS: MWF 11-11:30am, MW 1-2:15pm at 621 PGH.

If you e-mail me, please mention your course (1431) in the subject line.

Check your CASA account for quiz due dates; don’t miss any quizzes.

BUBBLE IN PS ID VERY CAREFULLY! If you make a bubbling mistake, your scantron will not be saved in the system and you will not get credit for it even if you turned it in. Bubble in Popper Number.

DID YOU TAKE PRACTICE TEST 3?

Be considerate of others in class. Respect your friends and do not distract anyone during the lecture.

POPPER#

Question# Which of the following functions is one-to-one?

A) \( f(x) = x^2 \)  B) \( f(x) = \sin(x) \)  C) \( f(x) = x^3 \)  D) \( f(x) = |x| \)  E) None
Chapter 4
Section 4.1 – Inverse Functions

RECALL—

**Definition:** A function is said to be **one-to-one** if there are no two distinct numbers in the domain of $f$ that produce the same value.

$$f(x_1) = f(x_2) \implies x_1 = x_2.$$ 

In other words, two different $x$ values cannot have the same $y$ value.

Which of the following functions are one-to-one? (Use the horizontal line test!)
**Definition:**

Let \( f \) be a one-to-one function. The inverse of \( f \), denoted as \( f^{-1} \), is the unique function defined on the range of \( f \) that satisfies the equation:

\[
f(f^{-1}(x)) = x, \text{ for all } x \text{ in the range of } f.
\]

We will use the notation \( f^{-1}(x) \) to denote the inverse of \( f(x) \).

Note: Domain of \( f \) becomes the range of \( f^{-1} \)

Range of \( f \) becomes the domain of \( f^{-1} \)

\[
f(a) = b \rightarrow f^{-1}(b) = a
\]

(a,b) is a point on the graph of \( f(x) \rightarrow (b,a) \) is a point on the graph of \( f^{-1}(x) \).

Example: Are these functions invertible?

\[
\begin{align*}
f(x) &= 5x - 1 \\
f(x) &= x^2 - x \\
f(x) &= x^5 + 4x
\end{align*}
\]

**How do we find the formula for the inverse of a function?**

1. Start with \( y = f(x) \).
2. **Switch** the x’s and y’s.
3. Solve for \( y \); get something like \( y = g(x) \).
4. The function \( g \) is the inverse of \( f \). \( g(x) = f^{-1}(x) \)

We can only do this for simple functions.
**Exercise:** Is \( f(x) = 2x - 5 \) invertible? If so, find its inverse.

**Exercise:** Find the inverse of \( f(x) = \frac{x + 2}{x - 1} \).

**How are functions related to their inverses?**

**Algebraic:**

**Geometric:**
NEW: A test for being invertible --

**Definition:** A function is **monotonic** if it is always increasing or always decreasing on its domain.

**Theorem:**
If \( f \) is monotonic, then \( f \) is an invertible function.

**Recall:**
If \( f' > 0 \), then \( f \) is increasing.
If \( f' < 0 \), then \( f \) is decreasing.
Hence, if \( f' > 0 \) on its domain, then \( f \) is monotonic.
If \( f' < 0 \) on its domain, then \( f \) is monotonic.

**Example:** Show that \( f(x) = x^3 + 5x \) is invertible.
Example: Let \( f(x) = \frac{1}{3}x^3 - x^2 + kx \). For what values of \( k \) is \( f(x) \) invertible?

Finding the Derivative of the Inverse Function

Theorem:

If \( f(x) \) is continuous and invertible then \( f^{-1}(x) \) is continuous.

Theorem:

If \( f(x) \) is differentiable and invertible, and \( f'(x) \) is nonzero, then \( f^{-1}(x) \) is differentiable.

Also, \( f(a) = b \) and \( f'(a) \neq 0 \), then \( (f^{-1})'(b) = \frac{1}{f'(a)} \).
**Example:** We showed earlier that $f(x) = x^3 + 5x$ was invertible.

Find $(f^{-1})'(6)$.

**Example:** Given $f(x) = x^5 + x + 2$, find $\left[ f^{-1} \right]'(36)$ if possible.
Example: Let $f(x) = x^5 + 2x^3 + 2x$. Give an equation for the tangent line to the graph of $f^{-1}(x)$ at the point $(-5, -1)$.

Example: Given $f(x) = 4x + \cos x - \sin x$ for $x \in \left[0, \frac{\pi}{2}\right]$, find $\left[f^{-1}\right]'(\pi)$ if possible. (Note: $f\left(\frac{\pi}{4}\right) = \pi$)
Example: If \( f \) is invertible, and \( f(1) = 2, f(3) = 1, f'(1) = 4, f'(3) = 5, f'(2) = 6 \), find \( (f^{-1})'(1) \).

Popper #

Q# If \( f(x) = x^3 + 2x \), find \( (f^{-1})'(12) \)?

a) 108 
b) 1/12 
c) 1/14 
d) -1/14 
e) None

Question# If \( f(x) = x^5 + x + 2 \), find \( (f^{-1})'(4) \)?

a. 6 b. 1/6 c. 1/4 d. 1/164 e. None