Popper#

Question# Give the derivative of $f(x) = \arcsin(e^{2x})$.

a. $f'(x) = \frac{2e^{2x}}{1 + e^{4x}}$

b. $f'(x) = \frac{2e^{2x}}{\sqrt{1 - e^{2x}}}$

c. $f'(x) = \frac{2e^{2x}}{\sqrt{1 - e^{4x}}}$

d. $f'(x) = \frac{e^{2x}}{\sqrt{1 - e^{4x}}}$

e. None
Question: Give the slope of the tangent line to the graph of
\[ f(x) = \arctan(4 - 2x) \text{ at } x = \frac{3}{2}. \]

a. -2
b. 2
c. 1/2
d. -1
e. None of the above
Section 4.5 – Hyperbolic Functions

Primary Definitions

Hyperbolic cosine: \[ \cosh(x) = \frac{e^x + e^{-x}}{2} \]

Hyperbolic sine: \[ \sinh(x) = \frac{e^x - e^{-x}}{2} \]

The four remaining hyperbolic functions are defined as you would expect given their names. That is:

\[ \tanh(x) = \frac{\sinh(x)}{\cosh(x)} \]
\[ \coth(x) = \frac{\cosh(x)}{\sinh(x)} \]
\[ \text{sech}(x) = \frac{1}{\cosh(x)} \]
\[ \text{csch}(x) = \frac{1}{\sinh(x)} \]
Derivatives of 6 basic hyperbolic functions:

\[
\frac{d}{dx} \sinh x = \cosh x \quad \frac{d}{dx} \cosh x = \sinh x
\]

\[
\frac{d}{dx} \tanh x = \sech^2 x \quad \frac{d}{dx} \sech x = -\tanh x \sech x
\]

\[
\frac{d}{dx} \coth x = -\csch^2 x \quad \frac{d}{dx} \csch x = -\coth x \csch x
\]

The six differentiation formulas are in the table below, written in chain rule form.

\[
\frac{d}{dx} \left( \sinh(u) \right) = \cosh(u) \frac{du}{dx} \quad \frac{d}{dx} \left( \coth(u) \right) = -\csch^2(u) \frac{du}{dx}
\]

\[
\frac{d}{dx} \left( \cosh(u) \right) = \sinh(u) \frac{du}{dx} \quad \frac{d}{dx} \left( \sech(u) \right) = -\sech(u) \tanh(u) \frac{du}{dx}
\]

\[
\frac{d}{dx} \left( \tanh(u) \right) = \sech^2(u) \frac{du}{dx} \quad \frac{d}{dx} \left( \csch(u) \right) = -\csch(u) \coth(u) \frac{du}{dx}
\]
Example: Differentiate: \( y = \cosh(5x) \)

Example: If \( f(x) = \sinh\left(x^2 + 4x\right) \); find \( f'(0) = ? \)

Example: Differentiate: \( y = \tanh\left(x^3\right) \)

Example: Differentiate: \( y = 5 \sech\left(2e^x\right) \)
Example: \( f(x) = 5\sinh(x) \); \( f''(x) = ? \)

Example:

\[
\frac{d}{dx} \left[ \frac{\cosh(x)}{1 + \sinh(x)} \right] =
\]
Example: Given \( f(x) = \cosh(2x) + \sinh^2 x \),

\( f(0) = ? \)

\( f'(0) = ? \)

Exercise: \( f(x) = \sinh(x + \cos(2x)) \); \( f'(0) = ? \)
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Q# Give the slope of the tangent line to

\[ f(x) = \cosh(2x) + \sinh(-3x) \] at \( x = 0 \).

a. \(-2\) b. \(2\) c. \(-1\) d. \(-3\) e. None of these

Q# \[ \frac{d}{dx} \left[ \cosh(2x) \right] = \]

a. \(\sinh(2x)\)  
b. \(- \cosh(2x)\)  
c. \(-2\sinh(2x)\)  
d. \(2\sinh(2x)\)  
e. None

Q# \[ \frac{d}{dx} \left[ \ln(\sinh x) \right] = \]

a. \(\sinh(x)\)  
b. \(- \cosh(x)\)  
c. \(\tanh(x)\)  
d. \(\coth(x)\)  
e. None
Q# \[ y = \cosh^2 x - \sinh^2 x, \quad y' = ? \]

a. \( 2\sinh(x) \)

b. \(-2 \cosh(x) \)

c. \( \cosh(2x) \)

d. 0

e. None
Chapter 5 Applications of Derivatives

Section 5.1 – Optimization

Optimization problems (to maximize or minimize):

1. Draw a picture, label it.
2. Determine the primary function (what is to be a max/min)
3. Use a secondary formula if necessary to get the primary function in terms of one variable.
4. Determine a feasible domain.
5. Find the max/min.
6. SHOW that the answer is a max/min using the First or Second Derivative test.

To maximize/minimize a function on a closed bounded interval, we evaluate the function at the endpoints, and then evaluate the function at any critical numbers in the interval.

Example 1: Find the dimensions to minimize the perimeter of a rectangular garden whose area is 60 square feet.
Example 2: A rectangular garden is to be fenced using 600 ft of fencing material. What are the dimensions that will maximize the area?

What is the largest possible area?