If you e-mail me, please mention your course (1431) in the subject line.

OFFICE HOURS: MWF 11-11:30am, MW 1-2:15pm at 621 PGH.

BBBLE IN PS ID VERY CAREFULLY! If you make a bubbling mistake, your scantron will not be saved in the system and you will not get credit for it even if you turned it in. Bubble in Popper Number.

Be considerate of others in class. Respect your friends and do not distract anyone during the lecture.

HOMEWORK: Read Section 6.1 from textbook before next lecture!
Chapter 6 – Integration

Section 6.1 – Definite Integral

How could we find the area under the curve of \( f(x) = x^2 \) and above the x-axis for \( x \in [0, 2] \) ?

The more rectangles we make, the more accurate the area!
**Definition:** A partition of a closed interval $[a,b]$ is a finite subset of $[a,b]$ that contains the points $a$ and $b$.

When we say $\{x_0, x_1, \ldots, x_n\}$ is a partition of $[a,b]$, we imply that $a = x_0 < x_1 < \ldots < x_n = b$.

For example, the sets $P_1 = \{0,1\}$, $P_2 = \left\{0, \frac{1}{2}, 1\right\}$, $P_3 = \left\{0, \frac{1}{10}, \frac{1}{2}, \frac{2}{3}, \frac{7}{8}, 1\right\}$ are partitions of the interval $[0,1]$.

If $P = \{x_0, x_1, \ldots, x_n\}$ is a partition of $[a,b]$, then $P$ breaks the interval to subintervals:

$$[x_0, x_1], [x_1, x_2], [x_2, x_3], \ldots, [x_{n-1}, x_n]$$

of lengths $\Delta x_1, \Delta x_2, \Delta x_3, \ldots, \Delta x_n$.

The lengths of these subintervals may or may not be equal.

If the lengths are equal, it is called a “**regular partition**” and $\Delta x = \frac{b-a}{n}$.

**Using Riemann Sums** –

Pick **any** point $x_i^*$ in the interval $[x_{i-1}, x_i]$.

This point $x_i^*$ may be

- the left endpoint of the interval $[x_{i-1}, x_i]$,
- the right endpoint of the interval $[x_{i-1}, x_i]$,
- the midpoint of the interval $[x_{i-1}, x_i]$,
- any other point in the interval $[x_{i-1}, x_i]$.

Now, using each point $x_i^*$, form the products:

$$f(x_1^*) \Delta x_1, f(x_2^*) \Delta x_2, \ldots, f(x_n^*) \Delta x_n$$
The sum

$$S^*(P) = f(x_1^*) \Delta x_1 + f(x_2^*) \Delta x_2 + \ldots + f(x_n^*) \Delta x_n$$

is called a Riemann Sum.

For each problem, APPROXIMATE the area under the curve using the given number and type of Riemann sums.

1. Left endpoint
   \[ f(x) = \frac{1}{x} \quad [1,2] \quad n = 4 \]

2. Right endpoint
   \[ f(x) = \frac{1}{x} \quad [1,2] \quad n = 4 \]
Example: Approximate the area under the curve over the given interval using Riemann Sums.

1. Left endpoint \( f(x) = \sqrt{x} \quad [1, 4] \quad n = 3 \)
2. Right endpoint \( f(x) = \sqrt{x} \quad [1,4] \quad n = 3 \)

3. Midpoint \( f(x) = \sqrt{x} \quad [1,4] \quad n = 3 \)
Upper Sum – Lower Sum

**Definition:** Let \( f \) be a continuous function on \([a, b]\) and \( P = \{x_0, x_1, ..., x_n\} \) be a partition of \([a, b]\).

The sum:

\[
U_f(P) = M_1 \Delta x_1 + M_2 \Delta x_2 + M_3 \Delta x_3 + ... + M_n \Delta x_n
\]

is called the **upper sum of** \( f \) with respect to the partition \( P \).

The sum:

\[
L_f(P) = m_1 \Delta x_1 + m_2 \Delta x_2 + m_3 \Delta x_3 + ... + m_n \Delta x_n
\]

is called the **lower sum of** \( f \) with respect to the partition \( P \).

1. Find an **Upper Sum** for \( f(x) = x^2, \ x \in [-1, 1] \) if the partition is

\[
P = \left[ -1, -\frac{3}{4}, \frac{1}{4}, \frac{1}{2}, 1 \right]
\]
2. Find a **Lower Sum** for \( f(x) = x^2, x \in [-1, 1] \) if the partition is

\[
P = \left[ -1, -\frac{3}{4}, \frac{1}{4}, \frac{1}{2}, 1 \right]
\]
Example: Find the upper sum over the interval [0,3] with n=3 rectangles.
Example: Find the lower sum over the interval [0,3] with n=3 rectangles.
Exercise: Find the upper and lower Riemann Sums:

\[ f(x) = 1 - x^2, \quad x \in [-1, 1] \text{ if the partition is } P = \left[-1, 0, \frac{1}{2}, 1\right] \]
For a function \( f \) which is continuous on \([a,b]\), there is one and only one number that satisfies the inequality

\[
L_f(P) \leq I \leq U_f(P),
\]

for all partitions \( P \) of \([a,b]\).

And that number is the number we use to define the definite integral.
**Definition:** Let \( f \) be continuous on \([a, b]\). The unique number that satisfies

\[
L_f (P) \leq I \leq U_f (P),
\]

for all partitions \( P \) of \([a, b]\)

is called the **definite integral** of \( f \) from \( a \) to \( b \) and is denoted by:

\[
\int_{a}^{b} f(x) \, dx.
\]

We read \( \int_{a}^{b} f(x) \, dx \) as: “the integral from \( a \) to \( b \) of \( f \) with respect to \( x \).”

The component parts have these names:

- \( \int \): the integral sign
- \( a \): lower limit of integration
- \( b \): upper limit of integration
- \( f(x) \): integrand.

The procedure of calculating the integral is called **integration**.

Notice that the “\( dx \)” is a part of the integral notation and it indicates the independent variable in discussion;

- \( \int_{a}^{b} f(x) \, dx \): the variable is \( x \),
- \( \int_{a}^{b} f(u) \, du \): the variable is \( u \).

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