Math 1431 DAY 35

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If you e-mail me, please mention your course (1431) in the subject line.

OFFICE HOURS: MWF 11-11:30am, MW 1-2:15pm at 621 PGH.

BUBBLE IN PS ID VERY CAREFULLY! If you make a bubbling mistake, your scantron will not be saved in the system and you will not get credit for it even if you turned it in. Bubble in Popper Number.

Be considerate of others in class. Respect your friends and do not distract anyone during the lecture.
Chapter 6 – Integration

Section 6.2 - The Fundamental Theorem of Calculus

Let \( f \) be a continuous function over the interval \([a,b]\). Define a new function by

\[
F(x) = \int_a^x f(t) \, dt.
\]

Here, the upper limit \( x \) varies between \( a \) and \( b \).

If \( f \) happens to be a nonnegative function, then \( F(x) \) can be seen as the “area under the graph of \( f \) from \( a \) to \( x \)”. We can think of \( F(x) \) as the “accumulated area” function.

**Theorem: Fundamental Theorem of Calculus Part 1**

If \( f \) is a continuous function over the interval \([a,b]\), then the function

\[
F(x) = \int_a^x f(t) \, dt
\]

is continuous on \([a,b]\) and differentiable on \((a,b)\). Moreover,

\[
F'(x) = \frac{d}{dx} \int_a^x f(t) \, dt = f(x), \quad \text{for all } x \text{ in } (a,b).
\]
17 \rightarrow \text{square} \rightarrow 17^2

5 \rightarrow 25

p \rightarrow \text{integrate} \rightarrow F

F \rightarrow \text{deriv.}
Example: $F(x) = \int_{1}^{x} ((t^2 + 2t) \, dt)$, \quad F'(x) = x^2 + 2x

\begin{align*}
\int_{a}^{x} F(x) \, dt &= \int_{1}^{\pi} 5 \cos(2t) \, dt = 0 \\
F'(4\pi) &= 5 \cdot \cos(8\pi) = 5 \\
F'(x) &= -\int_{1}^{x} \sin(2t) \, dt \\
F'(x) &= -\sin(2x)
\end{align*}
Example: $F(x) = \int_{0}^{5x^2} \frac{1}{1+t^2} \, dt$, $F'(x) =$

\[ F'(x) = \frac{1}{1 + (5x^2)^2} \cdot (5x^2)' \]

\[ = \frac{10x}{1 + 25x^4} \]

That is,

\[ \frac{d}{dx} \left( \int_{a}^{u(x)} f(t) \, dt \right) = f(u(x))u'(x). \]

Example: $F(x) = \int_{x^2}^{2x^3} (5t^2) \, dt$, $F'(x) =$

\[ F(x) = \int_{x^2}^{2x^3} (5t^2) \, dt + \int_{x^2}^{2x^3} (5t^2) \, dt \]

\[ F(x) = -\int_{a}^{x^2} (5t^{-2}) \, dt + \int_{a}^{2x^3} (5t^{-2}) \, dt \]

\[ F'(x) = -5(x^4)^{-2} \cdot 2x + 5(2x^3)^{-2} \cdot 6x^2 \]

\[ = -10x^6 + 120x^8 \]
Shortcut: \[
\frac{d}{dx} \left( \int_{u(x)}^{v(x)} f(t) \, dt \right) = f(v(x))v'(x) - f(u(x))u'(x)
\]

Exercise: \[
\frac{d}{dx} \left[ \int_{\sin x}^{2x} (1 + \sqrt{t}) \, dt \right],
\]

\[
= (1 + \sqrt{2x}) \cdot 2 - (1 + \sqrt{\sin x}) \cdot \cos x
\]

Example: Let \( f \) be a continuous function satisfying \( x^3 + x^2 - x = \int_1^x f(t) \, dt \).

Find \( f(x) \).

Find \( f'(x) \).

\[ F(x) = \frac{x^3 + x^2 - x}{x} \]

\[ F'(x) = f(x) = 3x^2 + 2x - 1 \]

\[ f'(x) = 6x + 2 \]
Clue: deriv of my function is 2x.

\[ \frac{1}{2} x^2 \]

\[ x^2 + \text{constant} \]

\[ x^2 + 1 \]

\[ x^2 + \sqrt{17} \]

\[ x^2 + C \quad c: \text{constant} \]
Exercise: For $x > 1$, let $F(x) = \int_{1}^{x} (t^2 - 6t) \, dt$.

When is $F$ increasing?

Find the critical numbers of $F$.

Study concavity of $F$.

Summary-

If we consider an integral as an “accumulation of area”, then the derivative of the integral is a “rate of change” of an “accumulation of an area”.

Therefore, if $F(x) = \int_{a}^{x} f(t) \, dt$, then $F'(x) = f(x)$.

- $F(a) = 0$
- Where $f$ is positive, $F$ is increasing
- Where $f$ is negative, $F$ is decreasing
- Where $f$ is zero, $F$ has possible max, min or inflection point
- Where $f$ is increasing, $F$ is concave up
- Where $f$ is decreasing, $F$ is concave down

HOMEWORK: READ Section 6.2 from your text book! Study the examples there.