Math 1431 DAY 37

Dr. Melahat Almus

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If you e-mail me, please mention your course (1431) in the subject line.

OFFICE HOURS: MWF 11-11:30am, MW 1-2:15pm at 621 PGH.

BBBLE IN PS ID VERY CAREFULLY! If you make a bubbling mistake, your scantron will not be saved in the system and you will not get credit for it even if you turned it in. Bubble in Popper Number.

Be considerate of others in class. Respect your friends and do not distract anyone during the lecture.
Chapter 6 – Integration

Section 6.2 - The Fundamental Theorem of Calculus

Theorem: Fundamental Theorem of Calculus Part 1

If \( f \) is a continuous function over the interval \([a,b]\), then the function

\[
F(x) = \int_a^x f(t) \, dt
\]

is continuous on \([a,b]\) and differentiable on \((a,b)\). Moreover,

\[
F'(x) = \frac{d}{dx} \int_a^x f(t) \, dt = f(x), \text{ for all } x \text{ in } (a,b).
\]

Definition: Let \( f \) be a continuous function over the interval \([a,b]\). A function \( G \) is called an antiderivative for \( f \) over the interval \([a,b]\) if \( G \) is continuous on \([a,b]\) and \( G'(x) = f(x) \) for all \( x \) in \((a,b)\).

Example: Find an antiderivative for

\[
f(x) = 2x^2 \quad x^2 \quad x^2 + 10c \quad x^2 + C
\]

\[
f(x) = 3x^2 \quad x^3 \quad x^3 + 1 \quad x^3 + C
\]

\[
f(x) = x^5 \quad \frac{x^6}{6} \quad + C
\]
Theorem: **Fundamental Theorem of Calculus Part 2**

Let $f$ be a continuous function over the interval $[a,b]$. If $G$ is any antiderivative for $f$ over the interval $[a,b]$, then

$$\int_a^b f(x) \, dx = G(b) - G(a).$$

**Example:** Calculate the definite integrals using FTOC.

$$\int_1^5 2x \, dx = \left[ x^2 \right]_{1}^{5} = 5^2 - 1^2 = 25 - 1 = 24$$

$G(x) : x^2$
\[ \int_{\pi/4}^{\pi/2} \cos x \, dx = \left[ \sin x + C \right]_{\pi/4}^{\pi/2} = \sin \left( \frac{\pi}{2} \right) + C - \left( \sin \left( \frac{\pi}{4} \right) + C \right) = 1 + C - \frac{\sqrt{2}}{2} - C = 1 - \frac{\sqrt{2}}{2} \]

\[ \int_{0}^{1} \left( 5x^4 + 4x^3 + 6 \right) \, dx = \left[ x^5 + x^4 + 6x \right]_{0}^{1} = 1 + 1 + 6 - 0 = 8 \]
\[ \int_{0}^{\frac{\pi}{4}} x \cos(x^2) \, dx \]

**Rule:**

\[ \int \cos(u) \, du = \sin(u) + C \]

\[ \int_{1}^{2} \ln(x) \, dx = \text{a number} \]
Section 6.3 – Basic Integration Rules

Indefinite integral:

The notation \( \int f(x) \, dx \) is used for an antiderivative of \( f \) and called an **indefinite integral**.

\[
\int f(x) \, dx = F(x) \quad \text{means} \quad F'(x) = f(x).
\]

In general, to find \( \int f(x) \, dx \), we find an antiderivative of \( f(x) \), say \( F(x) \), and then we write the indefinite integral as:

\[
\int f(x) \, dx = F(x) + C.
\]

Here, \( C \) is called the **constant of integration**.

Basic Rules of Integration

1) Power Rule for Integrals

\[
\int x^r \, dx = \frac{x^{r+1}}{r+1} + C, \quad \text{where} \quad r \neq -1.
\]

Examples:

\[
\int x^4 \, dx = \frac{x^5}{5} + C
\]

\[
\int x^{1/2} \, dx = \int x^{1/2} \, dx = \frac{x^{1/2+1}}{1/2+1} + C = \frac{x^{3/2}}{3/2} + C = \frac{2}{3} \cdot x^{3/2} + C
\]
\[
\int x^{-6} \, dx = \frac{x^{-6+1}}{-6+1} + C = -\frac{x^{-5}}{5} + C
\]

\[
\int \frac{1}{\sqrt{x}} \, dx = \int x^{-1/2} \, dx = \frac{x^{-1/2+1}}{-1/2} + C = \sqrt{x} + C
\]

2) \[\int k \cdot f(x) \, dx = k \int f(x) \, dx\], where \( k \) is a constant number.

3) \[\int [f(x) \pm g(x)] \, dx = \int f(x) \, dx \pm \int g(x) \, dx\].

4) \[\int k \cdot dx = k \cdot x + C\], where \( k \) is a constant number.

Using these rules, we can integrate \textbf{any polynomial}.

\[
\int 5 \, dx = 5x + C
\]

\[
\int 12 \, dx = 12x + C
\]

Example:

\[
\int (x^8 + 5x + 6 + 7) \, dx
\]

\[= \frac{x^9}{9} + 5 \cdot \frac{x^4}{4} + \frac{x^2}{2} + 7x + C\]
Example: \[ \int \frac{x^3 + 5x^3 + x + 4}{x^3} \, dx \]

\[ = \int x^2 + 5 + x^{-2} + 4x^{-3} \, dx \]

\[ = \frac{x^3}{3} + 5x + \frac{x^{-1}}{-1} + 4 \cdot \frac{x^{-2}}{-2} + C \]

Example: \[ \int \frac{x + 2\sqrt{x}}{\sqrt{x}} \, dx \]

Basic Formulas

Using the derivative formulas you learned in the previous chapters, we can derive several formulas for integration.

1) Integrals of Basic Trigonometric Functions:

\[ \int \sin x \, dx = -\cos x + C \]

\[ \int \cos x \, dx = \sin x + C \]

\[ \int \sec^2 x \, dx = \tan x + C \]

\[ \int \csc^2 x \, dx = -\cot x + C \]

\[ \int \sec x \tan x \, dx = \sec x + C \]

\[ \int \csc x \cot x \, dx = -\csc x + C \]
2) \( \int \frac{1}{x} \, dx = \ln|x| + C. \)

3) Integrals of Exponential Functions

\[
\int e^x \, dx = e^x + C \\
\int a^x \, dx = \frac{a^x}{\ln a} + C, \quad \text{where } a > 0, \ a \neq 1.
\]

4) Integrals Resulting in Inverse Trigonometric Functions

The following formulas are derived using the derivative formulas for inverse trigonometric functions (Chapter 4).

\[
\int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin x + C, \\
\int \frac{1}{1+x^2} \, dx = \arctan x + C, \\
\int \frac{1}{|x|\sqrt{x^2-1}} \, dx = \text{arc sec } x + C.
\]

5) Integrals of Hyperbolic Functions

\[
\int \sinh x \, dx = \cosh x + C \\
\int \cosh x \, dx = \sinh x + C
\]
### TABLE OF INTEGRALS

<table>
<thead>
<tr>
<th>Integral</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \int x^r , dx = \frac{x^{r+1}}{r+1} + C \quad r \neq -1 ]</td>
<td>[ \int \frac{1}{x} , dx = \ln</td>
</tr>
<tr>
<td>[ \int \sin x , dx = -\cos x + C ]</td>
<td>[ \int \cos x , dx = \sin x + C ]</td>
</tr>
<tr>
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<td>[ \int \csc x \cot x , dx = -\csc x + C ]</td>
</tr>
<tr>
<td>[ \int e^x , dx = e^x + C ]</td>
<td>[ \int a^x , dx = \frac{a^x}{\ln a} + C \quad a &gt; 0, a \neq 1 ]</td>
</tr>
<tr>
<td>[ \int \sinh x , dx = \cosh x + C ]</td>
<td>[ \int \cosh x , dx = \sinh x + C ]</td>
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#32 1-5: Mark A

Online office hour 8pm tonight