FINAL EXAM at CASA

Register as soon as possible.

Double check your date and time.

Have someone call you the day of the exam!

Don’t forget to bring your ID!

Approx. 20-22 Problems (Some MC, some FR)

Time: 110 minutes

Topic covered: Everything!

Practice Final: 5% is added to the final grade

Teacher Evaluations: +2 points on the final.

The percentage on the final (without any extra credit) will be used to replace one missed test OR the lowest test (if it is better). This will be done in a way that benefits you the most; if test 1 is your lowest test but replacing another test is more beneficial for your average, then that other test will be replaced. If you missed a test, that missed test will be replaced for sure.

Go over the class notes, work on past quizzes, EMCFs and practice tests.

We will solve some of these problems in class – the rest are exercises for you.

This review sheet is not a complete list of what you need to know. There may be questions on the final that are from topics not included in this sheet. Make sure you take the practice final (several times if necessary). This sheet should not be your only source while studying for the final. GO OVER THE REVIEW SHEETS (online and in class) FOR TESTS 2, 3, 4.
1. Find the following limits (if they exist):

a. \[ \lim_{{x \to 1}} \frac{\sqrt{x} - 1}{x - 1} = \lim_{{x \to 1}} \frac{\frac{1}{2\sqrt{x}}}{1} = \frac{1}{2} = \frac{1}{2} \]

b. \[ \lim_{{x \to 0}} \frac{\sin(5x)}{\sin(2x)} = \frac{5}{2} \]

c. \[ \lim_{{x \to 0}} \frac{1 - \frac{3}{x^2}}{4 + \frac{1}{x^2}} = \lim_{{x \to 0}} \frac{-3}{x^2} = \frac{6}{-2} = [-3] \]
d. \( \lim_{{x \to 0}} \frac{e^{2x} - 2x - 1}{1 - \cos(2x)} = \frac{2e^{2x} - 2}{2 \sin(2x)} = \frac{0}{0} \)

\[ \lim_{{x \to 0}} \frac{4e^{2x}}{4\cos(2x)} = \frac{e^0}{\cos(0)} = 1 \]

See Review for Test 4 for more L’Hospital’s rule questions!

2. Find values of A and B so that the function is continuous.

\[ f(x) = \begin{cases} 
  x^2 - B & x < 3 \\
  8 & x = 3 \\
  4x - 1 & x > 3 
\end{cases} \]

\[ f(3) = \lim_{{x \to 3^+}} f(x) = \lim_{{x \to 3^-}} f(x) = \frac{9 - B}{9 - B} \]

\[ 8 = 3A - 1 = 9 - B \]

\[ 3A - 1 = 8 \]

\[ 3A = 9 \]

\[ A = 3 \]

\[ 9 - B = 8 \]

\[ -B = -1 \]

\[ B = 1 \]
3. Find values of A and B so that the function is differentiable.

\[ f(x) = \begin{cases} 
  x^2 + A, & x < 1 \\
  Bx - 3, & x \geq 1
\end{cases} \]

Know the definition of the derivative.

\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}, \text{ provided the limit exists.} \]

Must be able to answer questions like: “Use the definition of derivative to find the derivative of \( f(x) = 3x^2 - x + 2 \).”
4. Find the derivative of the following:

a. \( f(x) = \frac{3}{x} + \tan(2x) \)
   \[ f'(x) = -\frac{3}{x^2} + 2\cdot\sec^2(2x) \]

b. \( y = \cos^3(2x) \)
   \[ y' = 3\cdot\cos^2(2x)\cdot[-2\cdot\sin(2x)] \]

c. \( f(x) = x\tan x \)
   \[ f'(x) = 1 + \tan x + x\cdot\sec^2(x) \]

d. \( f(x) = 5x\cos(2x) \)
   \[ f'(x) = 5\cdot\cos(2x) + 5x\cdot(-2\cdot\sin(2x)) \]

e. \( f(x) = \frac{2x+1}{4x-2} \)
   \[ f'(x) = \frac{2\cdot(4x-2)-(2x+1)\cdot4}{(4x-2)^2} = \frac{8x-4-8x-4}{(4x-2)^2} = \frac{-8}{(4x-2)^2} \]

f. \( f(x) = \sin(x^2+2x) \)
   \[ f'(x) = \cos(x^2+2x)\cdot(2x+2) \]

g. \( f(x) = x^3 + \frac{1}{x^2} \)
   \[ f'(x) = 3x^2 - \frac{1}{2x^3} \]

h. \( f(x) = \sec(3x) + \tan(5x^2) \)
   \[ f'(x) = 3\sec(3x)\cdot\tan(3x) + 10x\cdot\sec^2(5x^2) \]

i. \( f(x) = \sqrt{1+2\sin x} \)
   \[ f'(x) = \frac{1}{\sqrt{1+2\sin x}} \cdot (2\cos x) = \frac{\cos x}{\sqrt{1+2\sin x}} \]
j. \( f(x) = e^x + 4^{5x} + \sinh(10x) \)

\[
\begin{align*}
\frac{d}{dx}(f(x)) &= e^x + 4^{5x} \cdot 5 \cdot \ln(4) + 10 \cdot \cosh(10x) \\
\frac{d}{dx}(f(x)) &= 2x \cdot \sinh(x^2) + 10 \cdot \cosh(10x)
\end{align*}
\]

k. \( f(x) = \cosh(x^2) + \sinh(10x) \)

\[
\frac{d}{dx}(f(x)) = 2x \cdot \sinh(x^2) + 10 \cdot \cosh(10x)
\]

l. \( f(x) = \ln(x^4 + 1) \)

\[
\frac{d}{dx}(f(x)) = \frac{4x^3}{x^4 + 1}
\]

m. \( f(x) = \arcsin(x^3) + \arctan(4x) \)

\[
\frac{d}{dx}(f(x)) = \frac{1}{\sqrt{1-(x^3)^2}} \cdot 2x + \frac{1}{1+(4x)^2} \cdot 4
\]

5. If \( f(x) = 5x^3 + 2x + 1 \) is an invertible function, find the slope of the tangent line to \( f^{-1}(x) \) at the point where \( x = 8 \).

\[
\left[ f^{-1} \right]'(8) = \frac{1}{f'(a)} = \frac{1}{15 \cdot 8 + 2} = \frac{1}{122}
\]
Implicit Differentiation

6. Compute \( \frac{dy}{dx} \) and write the slope of the tangent lines to

a. \( y^2 - xy + 6 = 0 \) at the point \((5,2)\).

\[
2y \cdot y' - \left[ 1 \cdot y + x \cdot y' \right] = 0
\]

\[
2 \cdot 2 \cdot y' - \left[ 2 + 5 \cdot y' \right] = 0
\]

\[
4y' - 2 - 5y' = 0 \Rightarrow -y' = 0 \Rightarrow y' = -2
\]

b. \( 2y^3 - 5xy + y = -4x \) at the point \((3,1)\).

\[
6y^2 \cdot y' - 5 \cdot \left[ 1 \cdot y + x \cdot y' \right] + y' = -4
\]

\[
6y^2 \cdot y' - 5y - 5xy' + y' = -4
\]

\[
(6y^2 - 5x + 1)y' = -4 + 5y
\]

\[
y' = \frac{-4 + 5y}{6y^2 - 5x + 1}
\]

\[
\text{Slope at } (3,1) = \frac{-4 + 5}{6 - 5(3) + 1} = \frac{-1}{8}
\]
Exercise: Suppose we are given the data in the table about the functions $f$ and $g$ and their derivatives. Find the following values.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>$f'(x)$</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$g(x)$</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>$g'(x)$</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

a. $h(x) = f(x)g(x)$; find $h'(2) = ?$

b. $h(x) = \frac{x+1}{f(x)}$; find $h'(4)$
7. The graph given below is graph of $f'$. Determine where $f$ is increasing, decreasing, concave up, concave down:

- $f' > 0$ indicates $f$ is increasing.
- $f' < 0$ indicates $f$ is decreasing.
- $f'' > 0$ indicates concave up.
- $f'' < 0$ indicates concave down.

8. The value $x = a$ is a critical number for $f(x)$. Classify $a$ as a local maximum, local minimum or neither.

Given $f(x) = x^3 - 12x^2$, $x = 8$

- $f'(x) = 3x^2 - 24x$
- $f''(x) = 6x - 24$
- $f''(8) = 6\cdot8 - 24 = 24 > 0$

Local min. at $x = 8$. 
\[ f(1) = 2, \quad f'(1) = 0, \quad f''(1) = -4 \]

classify \quad x = 1.

\[ f''(1) < 0 \]

local max at \( x = 1 \).
9. Find the **absolute maximum/minimum value** for \( f(x) = x^2 - 4x + 1 \) on the interval \([-3, 3]\)

\[
\begin{align*}
\frac{d}{dx} f(x) &= 2x - 4 = 0 \\
\Rightarrow x &= 2 \\
\text{abs min value is } 22
\end{align*}
\]

10. Give the differential of \( f(x) = x^2 - 3x \) at \( x = 1 \) with respect to the increment \( 1/10 \).

11. Use differentials to approximate \( \sqrt{63} \).

\[
\begin{align*}
f(x) &= \sqrt{x} \\
\frac{df}{dx} &= \frac{1}{2\sqrt{x}} \\
df &= f'(64) \cdot h = \frac{1}{2\sqrt{64}} \cdot (-1) = \frac{-1}{16} \\
\sqrt{63} &\approx \sqrt{64} + df = 8 + \frac{-1}{16} = \frac{127}{16}
\end{align*}
\]
12. Evaluate:

a. \[
\frac{d}{dx} \int_0^x \sqrt{t^2 + 3} \, dt
\]

\[= \sqrt{(x^3)^2 + 3} \cdot 3x^2\]

b. \[F(x) = \int_0^x (2 \sin t) \, dt; \text{ Find } F'(\frac{\pi}{6}).\]

\[F'(x) = 2 \sin (4x) \cdot 4\]

\[F'(\frac{\pi}{6}) = 2 \cdot \sin \left(4 \cdot \frac{\pi}{6}\right) \cdot 4 = 8 \cdot \sin \left(\frac{2\pi}{3}\right)\]

\[= 8 \cdot \frac{\sqrt{3}}{2} = 4\sqrt{3}\]
13. Integrate:

a. \[ \int_{0}^{\pi} (x^{3} + 2\sqrt{x}) \, dx \]

\[ \int_{0}^{\pi} x^{1/2} \, dx \]

\[ = \left[ \frac{x^{3/2}}{3/2} \right]_{0}^{\pi} \]

\[ = \left( \frac{\pi^{3/2}}{3/2} \right) - (0) = \frac{\pi^{3/2}}{3/2} = \frac{\pi^{1.5}}{1.5} \]

b. \[ \int \left( e^{\ln x} + 5 \cdot \frac{1}{x} \right) \, dx \]

\[ = \frac{e^{4x}}{4} + \frac{5x}{\ln 5} + \ln |x| + C \]

c. \[ \int_{0}^{x} \frac{x}{x^{2} + 2} \, dx \]

Let \( u = x^{2} + 2 \)

\[ \frac{du}{dx} = 2x \]

\[ du = 2x \, dx \]

\[ \int \frac{x}{x^{2} + 2} \, dx = \int \frac{1}{u} \cdot \frac{du}{2} = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ln |u| + C \]

\[ = \frac{1}{2} \ln |9 + x^2| + C \]

d. \[ \int \sqrt{x^{2} + 2} \, dx \]

Let \( u = x^{2} + 2 \)

\[ \frac{du}{dx} = 2x \]

\[ du = 2x \, dx \]

\[ \int \sqrt{x^{2} + 2} \, dx = \int \sqrt{u} \, \frac{du}{2} = \frac{1}{2} \int u^{1/2} \, du \]

\[ = \frac{1}{2} \cdot \frac{u^{3/2}}{3/2} + C \]

\[ = \frac{1}{2} \cdot \frac{2x^{3/2}}{3} \cdot (x^{2} + 2)^{3/2} + C \]

\[ = \frac{1}{2} \left( x^{4/2} + 2 \right)^{3/2} + C \]
e. \[ \int \frac{2x + 1}{x^2 + x + 3} \, dx = \int \frac{1}{u^4} \, du \]

Let \( u = x^2 + x + 3 \)
\( \frac{du}{dx} = 2x + 1 \)

\[ du = (2x + 1) \, dx \]

\[ \int \frac{1}{u^4} \, du = \int u^{-4} \, du \]

\[ = \left[ \frac{u^{-3}}{-3} \right] \bigg|_3^5 \]

\[ = \frac{5^{-3} - 3^{-3}}{-3} \]

\[ = -\frac{5^{-3}}{3} + \frac{3^{-3}}{3} \]

f. \[ \int \sin(2x) \, dx = \int \sin(u) \cdot \frac{du}{2} \]

\( u = 2x \)
\( \frac{du}{dx} = 2 \)

\[ = \frac{1}{2} \sin(u) + C = \frac{1}{2} \sin(2x) + C \]

g. \[ \int \cos(3x) \, dx = \int \cos(u) \cdot \frac{du}{3} \]

\( u = 3x \)
\( \frac{du}{dx} = 3 \)

\[ = \frac{1}{3} \sin(u) + C = \frac{1}{3} \sin(3x) + C \]

h. \[ \int (\cos(2x) + \sec^2(x)) \, dx \]

\[ = \frac{1}{2} \sin(2x) + \tan(x) + C \]
\[ i. \int \sqrt{2x+1} \, dx = \int u \cdot \frac{du}{2} = \frac{1}{2} \int u^{3/2} \, du = \frac{1}{2} \cdot \frac{u^{3/2}}{3/2} + C = \frac{1}{3} (2x + 1)^{3/2} + C \]

\[ u = 2x + 1 \]
\[ \frac{du}{dx} = 2 \quad du = 2 \, dx \]

\[ j. \int x^4 (x^2 + 1)^4 \, dx = \int u^4 \cdot \frac{du}{2} \]

\[ u = x^2 + 1 \]
\[ \frac{du}{dx} = 2x \]

\[ k. \int e^u \cdot \frac{du}{2} = \frac{1}{2} \int e^u \, du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C \]

Let \( u = x^2 \)
\[ du = 2x \, dx \]

\[ l. \int \sinh(5x) \, dx = \frac{1}{5} \cosh(5x) + C \]

\[ m. \int \sec^2(10x) \, dx = \frac{1}{10} \tan(10x) + C \]

\[ n. \int \frac{16}{1 + x^2} \, dx = 16 \cdot \arctan(x) + C \]

\[ \text{formula} \]
\[ \int \frac{1}{1 + x^2} \, dx = \arctan(x) + C \]
These integrals may be asked as definite or indefinite – for ease of typing, I gave indefinite examples, but you need to know how to handle definite integrals as well.

* HW 14
* Friday's notes 3 important examples
14. Given \( f'(x) = 5e^x - 4 \) and \( f(0) = 2 \); find \( f(x) \).

\[
\begin{align*}
\int f'(x) \, dx &= \int (5e^x - 4) \, dx \\
&= 5e^x - 4x + C
\end{align*}
\]

Given: \( f(0) = 2 \)  \( \Rightarrow \)  

\[
\begin{align*}
f(0) &= 5e^0 - 4 \cdot 0 + C = 2 \\
5 + C &= 2 \\
C &= -3
\end{align*}
\]

\[f(x) = 5e^x - 4x - 3\]

\[
\begin{align*}
\int \sinh(x) \, dx &= \cosh(x) + C \\
\int \cosh(x) \, dx &= \sinh(x) + C
\end{align*}
\]

\[
\begin{align*}
\sinh(0) &= 0 \\
\cosh(0) &= 1
\end{align*}
\]

\[e^x: \quad f'(x) = 2 \sinh(x) + 3 \quad \text{\underline{integral}}\]

\[
f(x) = 2 \cosh(x) + 3x + C
\]

\[
f(0) = 2 \cdot 1 + 0 + C = 4 \quad \Rightarrow C = 2
\]
\[
\int_{0}^{6} f(x) \, dx = \int_{0}^{4} f(x) \, dx + \int_{4}^{6} f(x) \, dx
\]

\[
\int_{0}^{6} f(x) \, dx = 10 + (-3) = 7
\]
Given: \[ \int_{0}^{3} f(x) \, dx = 8 \]
\[ \int_{3}^{5} f(x) \, dx = -2 \]

Total area from \( x = 0 \) to \( x = 5 \) is \[ 8 + 2 = 10 \]
15. Given the graph of \( f(x) \).
With the area of region A equal to \( \frac{7}{3} \), region B is \( \frac{34}{3} \) and region C is \( \frac{7}{3} \).
Find:

\[ \text{area: } \frac{7}{3} + \frac{24}{3} + \frac{7}{3} \]

a. The area of the region bounded by \( f(x) \) and the x-axis between \(-2\) and \(4\)

b. \( \int_{-2}^{4} f(x) \, dx \)

\[ \int_{-2}^{4} f(x) \, dx = \int_{-2}^{-1} f(x) \, dx + \int_{-1}^{3} f(x) \, dx + \int_{3}^{4} f(x) \, dx \]

\[ = \frac{7}{3} + \left(-\frac{3}{4}\right) + \frac{7}{3} = \frac{20}{3} \]

16. Given: \[ \int_{1}^{8} f(x) \, dx = 10; \int_{1}^{2} f(x) \, dx = 3; \int_{4}^{8} f(x) \, dx = 4 \]. Find \( \int_{2}^{1} f(x) \, dx = ? \)

\[ \int_{1}^{8} f(x) \, dx = \int_{1}^{2} f(x) \, dx + \int_{2}^{4} f(x) \, dx + \int_{4}^{8} f(x) \, dx \]

\[ \int_{1}^{2} f(x) \, dx = \int_{1}^{2} f(x) \, dx + \int_{2}^{4} f(x) \, dx + \int_{4}^{8} f(x) \, dx \]

\[ \int_{1}^{8} f(x) \, dx = 3 \]

\[ \int_{-7}^{-7} f(x) \, dx = -3 \]
WORD PROBLEMS – Rate of change (3.1) and Optimization (5.3). Go over the examples we covered in class and online quiz questions! I included a few examples here but you need to practice the different types as well.

17. The base of a triangle is increasing at a rate of 2in/min. If the altitude is always twice the base, how fast is the area changing when the base is 20 inches?

18. Sand is falling into a cylindrical tank at a rate of 15 cubic feet per minute. The radius of the tank is 4 feet. At what rate is the height of the pile changing when it is 10 feet high?

19. A particle is moving on the curve $x^2y = 1$. As it passes through the point $(2,1/4)$, the x-coordinate is increasing at a rate of 2 units per second. At what rate is the y-coordinate changing at that time?

20. The length of a rectangle is increasing at a rate of 5 cm/sec and the width is decreasing at a rate of 2 cm/sec. At what rate is the area changing when length is 12cm and width is 4cm?

21. A rectangle is drawn in the first quadrant so that its base is on the x axis and its left side is on the y axis. What is the maximum area of this rectangle if its upper right vertex lies on the line segment connecting the points $(4,0)$ and $(0,8)$?
22. Suppose the velocity of a particle moving on the number line is given by \( v(t) = 6t^2 - 4t \) for \( t > 0 \). If the initial velocity is 0 and the initial position is the origin.

a) Find the acceleration when \( t = 2 \).

\[ a(t) = v'(t) = 12t - 4 \]

\[ a(2) = 24 - 4 = 20 \]

b) Find the position of this particle when \( t = 2 \).

\[ s(t) = \int v(t) \, dt = \int (6t^2 - 4t) \, dt = 2t^3 - 2t^2 + C \]

\[ s(0) = 0 \]

\[ C = 0 \]

\[ s(t) = 2t^3 - 2t^2 \]

\[ s(2) = 16 - 8 = 8 \]

23. Find two numbers whose sum is 10 and the sum of their squares is a minimum.
24. (EXERCISE) Consider the function \( f(x) = 3x^4 - 20x^3 + 42x^2 - 36x \).

a. Find the critical numbers of \( f \). Hint: \( f'(x) = 12(x - 3)(x - 1)^2 \)

b. Give the interval(s) of increase and decrease of \( f \).

c. Give the value(s) of \( x \) at which \( f \) has either a local minimum or a local maximum.

d. Give the interval(s) where the graph of \( f \) is concave up.

e. Give the values of \( x \) where the graph of \( f \) has inflection.
25. Compute the Riemann sums for the function \( f(x) = x^2 \) on the interval \([0, 2]\) associated with the partition \( P = \{0, 1, 2\} \).

a. left hand

\[
L_2 = f(0) \cdot 1 + f(1) \cdot 1 = 0 \cdot 1 + 1 \cdot 1 = 1
\]

b. right hand

\[
R_2 = f(1) \cdot 1 + f(2) \cdot 1 = 1 \cdot 1 + 4 \cdot 1 = 5
\]

c. midpoint

\[
M_2 = f(\frac{1}{2}) \cdot 1 + f(\frac{3}{2}) \cdot 1 = \frac{1}{4} + \frac{9}{4} \cdot 1 = \frac{5}{2}
\]

d. lower

\[
L_P = f(0) \cdot 1 + f(1) \cdot 1 = 0 + 1 = 1
\]

e. upper

\[
U_P = f(1) \cdot 1 + f(2) \cdot 1 = 1 + 4 = 5
\]

Must be able to answer “conceptual” questions about Riemann sum. For example, if the function is increasing which sum is the largest? Which is the smallest? On a decreasing function how do \( L_n \) and \( R_n \) compare? Etc.

ex: \( f \) is decreasing. \( L_4 \) \& \( R_4 \) compare

\[
L_4 > R_4
\]

\[
L_4 > \int f > R_4
\]
ex: f is increasing

\[
L_4 < \mathbb{R}_4
\]

\[
L_4 < \int f \leq \mathbb{R}_4
\]
Questions?

The graph of \( f(x) \) is given below and \( F(x) = \int_{0}^{x} f(t) \, dt \).

\[
\begin{align*}
\int_{0}^{1} f & = \text{area of } \square = 2.1 \cdot 2, \\
\int_{0}^{3} f & = \square + \triangle = 2 + \frac{1}{2} \cdot 2.2, \\
& = 2 + 2 = 4,
\end{align*}
\]

\[
\begin{align*}
\int_{0}^{5} f & = \int_{0}^{3} f + \int_{3}^{5} f = 4 + [-2] = 2
\end{align*}
\]
The diagram shows a region on the coordinate plane with boundaries at x = 0, x = 4, x = 5, and x = 6, and a point at (4, 5). The area of the region is given as \( \frac{1}{2} \) and the integral is negative.

The integral is split into two parts:

\[
\int_{0}^{7} f(x) \, dx = \int_{0}^{6} f(x) \, dx + \int_{6}^{7} f(x) \, dx
\]

The area of the region is calculated as:

\[
\left( 5 + 5 + 1 + \frac{1}{2} \right) + \left( -\frac{1}{2} \right) = 11
\]

The \( u \)-substitution review includes:

\[
\int x^2 (x^3 + 1)^5 \, dx
\]

Using \( u = x^3 + 1 \), \( du = 3x^2 \, dx \), and \( \frac{du}{3} = x^2 \, dx \), the integral becomes:

\[
\int u^5 \frac{du}{3} = \frac{1}{3} \int u^5 \, du = \frac{1}{3} \left( \frac{u^6}{6} \right) + C = \frac{1}{18} (x^3 + 1)^6 + C
\]
\[ \int \frac{4x}{x^2 + 5} \, dx = \int \frac{1}{u} \cdot 2 \, du \]
\[ u = x^2 + 5 \]
\[ du = 2x \, dx \]
\[ = 2 \ln |u| + C \]
\[ = 2 \ln |x^2 + 5| + C \]

\[ \text{Hint:} \quad \int \sin (10x) \, dx \]
\[ u = \text{inside} \]
\[ \int x \cdot \cos(2x^2) \, dx \]

\[ \int (2x) \, dx = x^2 + C \]
\[
\int x \cdot \sqrt{x^2 + 5} \, dx = \int \sqrt{u} \cdot \frac{du}{2}
\]

\(u = x^2 + 5\)

\(du = 2x \, dx\)

**Questions?**

- **critical** if \(f'(x) = 0\) or undefined
- **POI** if \(f''(x) = 0\) or undefined & sign change

U-sub video: Day 39, 40, 41.
\[ f^{\prime\prime}(x) = x^2 \cdot (x-2) \]

Possible POI: 0 & 2

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THE END!

Good luck!