Visit CASA regularly for announcements and course material!

If you e-mail me, please mention your course (1431) in the subject line.

Check online quiz due dates from CASA! No make ups.

Respect your friends! Do not distract anyone by chatting with people around you… Be considerate of others in class.

BUBBLE IN PS ID VERY CAREFULLY! If you make a bubbling mistake, your scantron will not be saved in the system and you will not get credit for it even if you turned it in.

Bubble in Popper Number.

POPPER#  0 1

Question# 1 State whether it is possible or not to create such a function:

- The function $f$ is continuous on $[1,4]$.
- It takes on values -2 and 5, but it does not take on the value 0.

A) Possible  B) Not possible
Question # 2: State whether it is possible or not to create such a function:

- The function $f$ is defined on $[1,4]$.
- $f$ is continuous on $(1,4)$.
- It takes only two distinct values.

A) Possible  B) Not possible

Question # 3: Compute $\lim_{x \to 0} \frac{\sin(8x)}{2x} = \frac{8}{2}$

a) 8  b) 4  c) 2  d) 0  e) DNE

Question # 4: $\lim_{x \to \pi} \frac{\sin(5x)}{2x} = \frac{5}{2}$

a) $5/2$  b) $2/5$  c) $0$  d) 1  e) DNE
Section 1.6 – Pinching Theorem, Limits of Trig Functions

In general,
\[
\lim_{x \to c} \sin(x) = \sin(c) \quad \text{and} \quad \lim_{x \to c} \cos(x) = \cos(c).
\]

Fact:
\[
\lim_{x \to 0} \frac{\sin(x)}{x} = 1 \quad \text{and} \quad \lim_{x \to 0} \frac{1 - \cos(x)}{x} = 0.
\]

Fact:
\[
\lim_{x \to 0} \frac{\sin(ax)}{bx} = \frac{a}{b} \quad \text{and} \quad \lim_{x \to 0} \frac{1 - \cos(ax)}{bx} = 0.
\]

ALWAYS pay attention to what value \(x\) is approaching.

Examples:
\[
\lim_{x \to 0} \frac{\sin(4x)}{\sin(9x)} = \lim_{x \to 0} \frac{\sin(4x)}{x} \cdot \frac{x}{\sin(9x)} = \frac{4}{9}.
\]
\[
\lim_{x \to 0} \frac{\sin^2(4x)}{5x} = \lim_{x \to 0} \frac{\sin(4x)}{5x} \cdot \frac{\sin(4x)}{1} = \frac{4}{5} \cdot \sin(4 \cdot 0) = \frac{4}{5} \cdot 0 = 0.
\]
\[ \lim_{x \to 0} \frac{\sin^2(4x)}{5x^2} = \lim_{x \to 0} \frac{\sin(4x)}{5x} \cdot \frac{\sin(4x)}{x} = \frac{4}{5} \cdot \frac{4}{4} = \frac{16}{5} \]

\[ \lim_{x \to \pi/2} \frac{\sin(2x)}{6x} = \text{by fact} \]

\[ \lim_{x \to 0} \frac{1 - \cos(2x)}{5x} = 0 \]

\[ \lim_{x \to 0} \frac{1 - \cos(4x)}{9x} = 0 \]

\[ \lim_{x \to \pi} \frac{1 - \cos(3x)}{6x} = \frac{1 - \cos(3\pi)}{6\pi} = \frac{1 - (1)}{6\pi} = \frac{2}{6\pi} = \frac{1}{3\pi} \]
Examples:

\[
\lim_{x \to 0} \frac{\tan(5x)}{4x} = \lim_{x \to 0} \frac{\sin(5x)}{\cos(5x)} \cdot \frac{5}{4} = \lim_{x \to 0} \frac{\sin(5x)}{4x \cdot \cos(5x)} = \frac{5}{4} \cdot \frac{1}{\cos(0)} = \frac{5}{4}
\]

\[
\lim_{x \to 0} \frac{\tan(6x)}{x \cdot \tan(4x)} = \frac{6}{4} = \frac{3}{2}
\]

\[
\lim_{x \to 0} \frac{\tan^2(4x)}{x^2} = \lim_{x \to 0} \frac{\tan(4x)}{x} \cdot \frac{\tan(4x)}{x} = \lim_{x \to 0} \frac{\tan(4x)}{x} \cdot \frac{\tan(4x)}{x} = \frac{4}{1} \cdot \frac{4}{1} = 16
\]

Example:

\[
\lim_{x \to 0} \frac{1 - \cos^2(2x)}{7x} = \lim_{x \to 0} \frac{\sin^2(2x)}{7x} = \lim_{x \to 0} \frac{\sin(2x)}{7x} \cdot \sin(2x) = \frac{2}{7} \cdot \sin(0) = 0
\]
\[
\lim_{x \to 0} \frac{\sin(ax)}{bx} = \frac{a}{b} \\
\lim_{t \to 0} \frac{\sin(at)}{bt} = \frac{a}{b}
\]

Example: \( \lim_{x \to 0} x^2 \csc(5x^2) = \lim_{x \to 0} \frac{x^2}{\sin(5x^2)} = \frac{\sqrt{5}}{5} \)

\[
\frac{1-1}{0} = \frac{0}{0} \\
\text{Example: } \lim_{x \to 0} \frac{1 - \cos(x)}{4x^2} = \frac{1}{4}
\]

\[
\lim_{x \to 0} \frac{(1 - \cos(x))}{4x^2} \cdot \frac{(1 + \cos(x))}{(1 + \cos(x))} = \lim_{x \to 0} \frac{\sin^2(x)}{4x^2} \cdot \frac{1}{1 + \cos(x)}
\]

\[
\lim_{x \to 0} \frac{\sin^2(x)}{4x^2} = \lim_{x \to 0} \frac{\sin(x)}{x} \cdot \frac{\sin(x)}{x} = \frac{1}{1 + \cos(x)}
\]
\[
\frac{1}{4} \cdot 1 \cdot \frac{1}{1 + \cos(0)} = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}
\]
Example: \[ \lim_{x \to 0} \frac{\sin^2(x) \csc(x)}{2x} = \lim_{x \to 0} \frac{\sin(x)}{2x} = \frac{1}{2} \]

Question# \[ \lim_{x \to \pi} \frac{\tan(6x)}{2x} = \]

a) 0 b) 6 c) 3 d) 1 e) DNE
The graph of the function $y = x^3 - x^2 + x - 1$ is shown. The tangent line at the point $(2, f(2))$ is indicated. The value of $f(2)$ is not explicitly calculated in the image.
Chapter 2

Section 2.1 – The Definition of the Derivative

What is a tangent line?

What is a secant line?
Goal: Given a function, find the slope of the tangent line at the point \((a, f(a))\).

Let’s try to “approximate” the slope of the tangent line at the point \((a, f(a))\).
Pick another point $P(a + h, f(a + h))$ on the curve and draw the secant line:

The slope of this secant line is:
$$\frac{f(a + h) - f(a)}{a + h - a} = \frac{f(a + h) - f(a)}{h}.$$ This expression is called the difference quotient of $f$ at $(a, f(a))$.

To make a better approximation, we need to move the two points closer together. In other words, we need to make $h$ smaller.
Now, the slope of this new secant line is a better approximation for the slope of the tangent line. We want to let the distance between two points get smaller and smaller in order to minimize the error. That is, we need to let $h$ approach 0 by taking limits. This gives us a formula for finding the slope of the tangent line.

Check this link:  
http://www.geogebratube.org/student/m11426

The slope of a tangent line to a function $f(x)$ at the point $(a, f(a))$ is

$$
\lim_{{h \to 0}} \frac{f(a + h) - f(a)}{h} = \text{slope of tangent line at } (a, f(a))
$$

if this limit exists.
**Definition:** A function $f$ is said to be **differentiable at** $a$, if the limit

$$\lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

exists.

The value of this limit is called the **derivative of** $f$ **at** $a$ and is denoted by $f'(a)$.

In other words, we define

$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

provided that the limit exists.

Note that $f'(a)$ is a real number and gives the slope of the tangent line at $(a, f(a))$. This number is also said to be the **slope of the graph of** $f$ **at** $(a, f(a))$. 
Remarks: 1. Since \( f'(a) \) is the slope of the tangent line at \((a, f(a))\), using the point-slope equation of a line, we can write the equation of the tangent line:

\[
y - f(a) = f'(a)(x - a).
\]

2. \( f'(a) \) is often referred to as the \textbf{rate of change} in \( f(x) \) at \( x = a \).

Example: Find the slope of the line tangent to the function \( f(x) = x^2 \) at the point \((3, 9)\). Write the equation of the tangent line.

\( \text{on } \underline{\text{Day 7 Notes}} \).