If you e-mail me, please mention your course (1431) in the subject line.

**Be considerate of others in class. Respect your friends and do not distract anyone during the lecture.**

Check your CASA account for quiz due dates; don’t miss any quizzes.

Purchase popper scantrons from UH Bookstore. Bring one scantron to every class.

**BBBLE IN PS ID VERY CAREFULLY! If you make a bubbling mistake, your scantron will not be saved in the system and you will not get credit for it even if you turned it in.**

**Bubble in Popper Number.**

**POPPERS**

<table>
<thead>
<tr>
<th>Question#</th>
<th>( \lim_{x \to 0} \frac{\tan^2(6x)}{2x^2} = )</th>
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<tbody>
<tr>
<td>a) 0</td>
<td>b) 18</td>
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Chapter 2

Section 2.1 – The Definition of the Derivative

Goal: Given a function, find the slope of the tangent line at the point \((a, f(a))\).

**Definition:** A function \(f\) is said to be **differentiable at** \(a\), if the limit

\[
\lim_\limits{h \to 0} \frac{f(a + h) - f(a)}{h}
\]

exists.

The value of this limit is called the **derivative of** \(f\) **at** \(a\) and is denoted by \(f'(a)\).

In other words, we define

\[
f'(a) = \lim_\limits{h \to 0} \frac{f(a + h) - f(a)}{h},
\]

provided that the limit exists.
Note that \( f'(a) \) is a real number and gives the slope of the tangent line at \((a, f(a))\). This number is also said to be the slope of the graph of \( f \) at \((a, f(a))\).

**Remarks:**

1. Since \( f'(a) \) is the slope of the tangent line at \((a, f(a))\), using the point-slope equation of a line, we can write the equation of the tangent line:
   \[ y - f(a) = f'(a)(x - a). \]

2. \( f'(a) \) is often referred to as the rate of change in \( f(x) \) at \( x = a \).

**Question#**

Given \( f(x) \) is a differentiable function, what does \( f'(1) \) represent?

a) The derivative of \( f \) at \( x = 1 \).
b) The slope of the tangent line at \( x = 1 \).
c) The rate of change of \( f \) at \( x = 1 \).
d) All of the above
e) None of these.
Example 1: Find the slope of the line tangent to the function \( f(x) = x^2 \) at the point \((3,9)\). Write the equation of the tangent line.

Exercise: Find \( g'(1) \) given that

\[ g(x) = \sqrt{x}. \]
The Derivative as a Function

Definition:

Given a function \( f \), the **derivative** of \( f \) is the function \( f' \) defined as:

\[
f'(x) = \lim_{{h \to 0}} \frac{f(x + h) - f(x)}{h}, \quad \text{provided the limit exists.}
\]

The domain of \( f'(x) \) is the set of all points where the defining limit exists, that is, all \( x \) for which \( f \) is **differentiable**.

To **differentiate** a function means to find its derivative.

**Remark:** 1. To apply this definition, \( f \) must be defined at some open interval containing \( x \). It is important to also note that when taking the limit, the variable is \( h \) and \( x \) is fixed.

2. Wherever \( f'(x) \) is defined, it is the slope of the graph of \( f \) at \((x, f(x))\) and it also gives the rate of change in \( f(x) \).
Example: Given \( f(x) = x^2 + 5x + 1 \), find \( f'(x) \) using the definition.

Exercise: Find the derivative of \( f(x) = x^2 + 5x + 1 \).

Exercise: Find the derivative of \( f(x) = \sqrt{x + 2} \).

Exercise: Find the derivative of \( f(x) = \frac{1}{x + 5} \).
Differentiability

A function $f$ is **differentiable at $a$** if

$$\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

exists. If this limit fails to exist, we say that the function is not differentiable at $a$.

If a function is differentiable at every number in an open interval $I$, we say that the function is **differentiable on $I$**. For example, the function $f(x) = \sqrt{x}$ is differentiable on the interval $(0, \infty)$.

**What is the relation between continuity and differentiability?**

**Fact:** If $f$ is differentiable at $a$, then it is continuous at $a$.

However, the converse of this statement is not always true. A function can be continuous at $a$ without being differentiable. For example, $f(x) = |x|$ is continuous at every real number but it is not differentiable at 0.
When is a function not differentiable at a point?

- The first problem that makes a function $f$ not differentiable at $a$ is discontinuity at $a$.

- Another issue is having a “sharp corner”, a cusp, or a vertical tangent.

The first graph is not differentiable at 1 since one-sided derivatives do not coincide. The left-hand derivative at 1 is -1, but the right-hand derivative is 0. We may refer to this case as $f$ having a *sharp corner* at 1. On the second graph, there is a *vertical tangent* at 0. And, on the last graph there is a *cusp* at 0. We will discuss vertical tangents and cusps in detail later.
Example: Is this function differentiable at $x=1$?

$$f'(x) = \begin{cases} 2x, & \text{if } x > 1 \\ 5x - 3, & \text{if } x \leq 1 \end{cases}$$

Check:

1) Is $f$ continuous at $x=1$? (If not, can’t be differentiable!)

2) Do we have: “Right derivative at $x=1$ equals the left derivative at $x=1$”? 

Right derivative: $f'_{+}(1) = 2$

Left derivative: $f'_{-}(1) = 5$

Observe the graph:

We will discuss this further after covering the rules of differentiation…
Section 2.2 – Derivatives of Polynomials and Trig Functions

Fact: If \( f(x) = k \), where \( k \) is a real number, then \( f'(x) = \).

Fact: If \( f(x) = x \), then \( f'(x) = \)

Now, here’s the formula to find the derivatives of power functions:

Fact: Power Rule

If \( f(x) = x^n \), where \( n \) is any positive integer, then \( f'(x) = nx^{n-1} \).

Examples:

\[ f(x) = x^2; \quad f'(x) = \]

\[ f(x) = x^3; \quad f'(x) = \]

\[ f(x) = x^4; \quad f'(x) = \]
\[ f(x) = \frac{1}{x}; \quad f'(x) = \]

\[ f(x) = \frac{1}{x^2}; \quad f'(x) = \]

**Theorem:** Let \( k \) be a real number. If \( f \) and \( g \) are differentiable at \( x \), then so are \( f + g \), \( f - g \) and \( k \cdot f \). Moreover,

- \((f \pm g)'(x) = f'(x) \pm g'(x)\),
- \((k \cdot f)'(x) = k \cdot f'(x)\).

**Theorem: Derivatives of Polynomials**

Let \( f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0 \) be a polynomial function. The derivative is:

\[ f'(x) = a_n \cdot n \cdot x^{n-1} + a_{n-1} \cdot (n-1) \cdot x^{n-1} + \ldots + a_2 \cdot 2x + a_1. \]
Examples:
\[ f(x) = x^3 + x^2 + x + 2; \quad f'(x) = \]
\[ f(x) = 5x^{10} - 6x^3 + 12; \quad f'(x) = \]

Example:
\[ f(x) = 2x^5 + x^4 - 5x + 6; \quad f'(1) = \]

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Question# \[ f(x) = x^3 + 2x + 5, \quad f'(0) = ? \]

a) 5  b) 3  c) 4  d) 2  e) 0

Question#
If \[ f(1) = 5 \text{ and } f'(1) = 6, \] what is the equation of tangent line at \( x = 1 \)?

a) \( y = 5x + 6 \)
b) \( y = 6x+5 \)
c) \( y= 6x -1 \)
d) \( y= 6x+1 \)
e) None