Math 1431 DAY 8

Dr. Almus

almus@math.uh.edu

Office hours: MWF 11-11:30am, MW: 1-2:15pm at 621 PGH.

If you e-mail me, please mention your course (1431) in the subject line.

Be considerate of others in class. Respect your friends and do not distract anyone during the lecture.

Check your CASA account for quiz due dates; don’t miss any quizzes.

Purchase popper scantrons from UH Bookstore. Bring one scantron to every class.

BUBBLE IN PS ID VERY CAREFULLY! If you make a bubbling mistake, your scantron will not be saved in the system and you will not get credit for it even if you turned it in.

Bubble in Popper Number.

Popper #

Question#  If \( f(0) = 2 \) and \( f'(0) = 4 \), what is the equation of the tangent line at \( x = 0 \)?

a) \( y = 2x - 4 \)

b) \( y = 4x - 2 \)

c) \( y = 4x + 2 \)

d) \( y = 2x + 4 \)

e) None
Question# Is $f(x) = \frac{1}{x^2}$ differentiable at $x = 0$?

a) Yes  b) No

Question# $f(x) = x^3 + x^2 + 2$, $f''(1) =$?

a) 3  
b) 4  
c) 5  
d) 6  
e) None

Recall--- Section 2.2 – Derivatives of Polynomials and Trig Functions

Fact: Power Rule

If $f(x) = x^n$, where $n$ is any positive integer, then $f'(x) = nx^{n-1}$.

Theorem: Let $k$ be a real number. If $f$ and $g$ are differentiable at $x$, then so are $f + g$, $f - g$ and $k \cdot f$. Moreover,

- $(f \pm g)'(x) = f'(x) \pm g'(x)$,
- $(k \cdot f)'(x) = k \cdot f'(x)$. 

Theorem: Derivatives of Polynomials

Let \( f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0 \) be a polynomial function. The derivative is:

\[
f'(x) = a_n \cdot n \cdot x^{n-1} + a_{n-1} \cdot (n - 1) \cdot x^{n-2} + \ldots + a_2 \cdot 2x + a_1.
\]

Example:

\[
f(x) = 2x^5 + x^2 - \frac{1}{2} x^2 + 6x + 4; \quad f'(x) =
\]

What if the function is not a polynomial?

Example:

\[
f(x) = 4x^3 + 5x + \frac{1}{x} + \frac{2}{x^2}; \quad f'(x) =
\]

Example:

\[
f(x) = \frac{x^5 + 10x^3 + 4}{x^2}; \quad f'(x) =
\]
Tangent and Normal lines:

Slope of the tangent line at \( x = a \) = Derivative of the function at \( x = a \).

\[
m_{\text{tangent}} = f'(a) \quad \text{and} \quad m_{\text{normal}} = -\frac{1}{m_{\text{tangent}}} = -\frac{1}{f'(a)}.\]
Example: Given: \( f(x) = x^3 - 2x^2 + 3x \)

a) Find the equation of the line \textit{tangent} to the curve at \( x = 2 \).

b) Find the equation of the line \textit{normal} to the curve at \( x = 2 \).
Example: Consider the function $f(x) = 3x^4 - 6x^2 + 5$. Find the points where the tangent line is horizontal.

Example: Consider the function $f(x) = x^2 + x + 2$. Find the points where the normal line has slope 4.
Derivatives of the Six Trig Functions:

\[
\begin{align*}
(sin x)' &= cos x \\
(cos x)' &= -sin x \\
(tan x)' &= \sec^2 x \\
(cot x)' &= -\csc^2 x \\
(\sec x)' &= \sec x \cdot tan x \\
(\csc x)' &= -\csc x \cdot cot x
\end{align*}
\]

Example: For \( f(x) = 10\sin x \), find \( f'\left(\frac{\pi}{6}\right) \).

Example: For \( f(x) = 2\cos x + 4\sin x \), find \( f'\left(\frac{\pi}{4}\right) \).

Example: For \( f(x) = 6\tan x - \sin x \), find \( f'(x) \).
**Example:** Consider the function \( f(x) = 2\sin x \), find the points where the tangent line is horizontal.

**Example:** Consider the function \( f(x) = 4\cos x \) over \([0,2\pi]\), find the points where the **tangent** line has slope 2.
Leibniz’s d/dx Notation

The derivative of a function $y$ can be denoted as
\[ \frac{dy}{dx}, \text{ if } y \text{ is a function is in terms of } x, \]
\[ \frac{dy}{dt}, \text{ if } y \text{ is a function is in terms of } t, \]
and so on. Here, $\frac{dy}{dx}$ indicates the derivative of $y$ with respect to $x$.

If $y = x^2$, then $\frac{dy}{dx} = $.

If $y = t^2$, then $\frac{dy}{dt} = $.

If $w = 5z^3 + z$, then we may take the derivative of $w$ with respect to $z$: $\frac{dw}{dz} = $.

The double-$d$ notation may also be used as a prefix to the function to be differentiated:
\[ \frac{d}{dx}(\text{expression}) = \text{the derivative of "expression" with respect to } x. \]

Example: \[ \frac{d}{dx}\left(x + \frac{1}{x}\right) = ? \]
Example: \( \frac{d}{dt}(5\cos t) = ? \)

Example: If \( y = x^3 + x \), find \( \frac{dy}{dx} \) at \( x = 2 \).

**POPPER #**

**Question#** \( f(x) = 5x + \frac{1}{x} \), what is the slope of the tangent line at \( x = 1 \)?

a) 6  
b) 4  
c) 5  
d) -1/5  
e) None

**Question#** \( f(x) = 5x^2 \); what is the slope of the normal line at \( x = 2 \)?

a) 10  
b) -1/10  
c) 1/10  
d) -1/20  
e) None