Math 1431

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Be considerate of others in class. Respect your friends and anyone during the lecture.

Check your CASA account for quiz due dates; don’t miss any quizzes.

BUBBLE IN PS ID VERY CAREFULLY! If you make a bubbling mistake, your scantron will not be saved in the system and you will not get credit for it even if you turned it in.

Bubble in Popper Number.

Popper #

Question# 1
If \( f(x) = \frac{5x^3 + x^2 + x}{x^2} \), \( f'(1) = ? \)

a) 7 
 b) 5 
 c) 4 
 d) 2 
 e) None

\[ f(x) = 5x + 1 + \frac{1}{x} \]

\[ f'(x) = 5 \sin x + 4 \cos x \]

Question# 2
If \( f(x) = 5 \cos x + 4 \sin x \), \( f'(\pi) = ? \)

a) -5 
 b) 4 
 c) -4 
 d) 1 
 e) None

\[ f'(x) = -5 \sin x + 4 \cos x \]
Question# If \( f(x) = \tan x + 6\cot x \), \( f''\left(\frac{\pi}{4}\right) = ? \)

a) -10  
b) 7  
c) -12  
d) 14  
e) None

Recall--

Power Rule

If \( f(x) = x^n \), where \( n \) is any positive integer, then \( f'(x) = nx^{n-1} \).

Slope of the tangent line at \( x = a \) = Derivative of the function at \( x = a \).

\[ m_{\text{tangent}} = f'(a) \quad \text{and} \quad m_{\text{normal}} = -\frac{1}{m_{\text{tangent}}} = -\frac{1}{f'(a)}. \]

Derivatives of the Six Trig Functions:

\[
\begin{align*}
(sin x)' &= \cos x \\
(cos x)' &= -\sin x \\
(tan x)' &= sec^2 x \\
(cot x)' &= -csc^2 x \\
(sec x)' &= sec x \cdot tan x \\
(csc x)' &= -csc x \cdot cot x
\end{align*}
\]
Higher Order Derivatives:

\[ f(x) = x^4 + x^3 + x^2 + 2x + 10 \]

The first derivative of \( f \) : 
\[ f'(x) = 4x^3 + 3x^2 + 2x + 2 \]

The second derivative of \( f \) : 
\[ f''(x) = \left[ f'(x) \right]' = 12x^2 + 6x + 2 \]

The third derivative of \( f \) : 
\[ f'''(x) = \left[ f''(x) \right]' = 24x + 6 \]

The fourth derivative of \( f \) : 
\[ f^{(4)}(x) = \left[ f'''(x) \right]' = 24 \]

The fifth derivative of \( f \) : 
\[ f^{(5)}(x) = \left[ f^{(4)}(x) \right]' = 0 \]

In general, \( f^{(n)} \) stands for the \( n \)th order derivative of \( f \).

The functions \( f', f'', f''', f^{(4)}, \ldots, f^{(n)} \) are called the derivatives of \( f \) of orders 1, 2, 3, \ldots, \( n \), respectively.

Remark:

\( f^{(4)}(x) \) stands for the fourth order derivative of \( f(x) \), while \( f^4(x) \) means \( \left[ f(x) \right]^4 \).

To see a variant of this notation, let \( y = x^5 \):

\[ y' = 5x^4, \quad y'' = 20x^3, \quad y''' = 60x^2, \quad \text{and so on.} \]
With the **double-\(d\) notation**, the second order derivative is:

\[
\frac{d^2 y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) \quad \text{or} \quad \frac{d^2}{dx^2}[f(x)] = \frac{d}{dx}\left[\frac{d}{dx}[f(x)]\right],
\]

and the third order derivative is:

\[
\frac{d^3 y}{dx^3} = \frac{d}{dx}\left(\frac{d^2 y}{dx^2}\right) \quad \text{or} \quad \frac{d^3}{dx^3}[f(x)] = \frac{d}{dx}\left[\frac{d^2}{dx^2}[f(x)]\right],
\]

and so on.

**Example:** \(f(x) = x^4 - x^2 + 5x; \quad f''(1) = ?\)

\[
f'(x) = 4x^3 - 2x + 5
\]

\[
f''(x) = 12x^2 - 2
\]

\[
f''(1) = 12(1)^2 - 2 = 10
\]

**Example:** \(f(x) = x^5 + x; \quad \frac{d^2 f}{dx^2} = ?\)

\[
f'(x) = 5x^4 + 1
\]

\[
f''(x) = 20x^3
\]
Example: \( y = 10 \cos t \); \( \frac{d^3 y}{dt^3} = ? \)

\[
\begin{align*}
\frac{dy}{dt} &= -10 \sin t \\
\frac{d^2 y}{dt^2} &= -10 \cos t \\
\frac{d^3 y}{dt^3} &= 10 \sin t
\end{align*}
\]

\[
\left( u \cdot v \right)' = \frac{u'}{v} - \frac{u}{v'}
\]

\[
f(x) = (x^5 + x^3 - x^2) \cdot (10x^7 - x^6 + x)
\]

\[
f'(x) = ?
\]
Section 2.3 – Differentiation Rules

Theorem: The Product Rule

If \( f \) and \( g \) are differentiable at \( x \), then so is the product \( fg \). Moreover,

\[
(fg)'(x) = f'(x)g(x) + f(x)g'(x)
\]

This formula may be written as:

\[
(uv)' = u'v + uv' \quad \text{or} \quad \frac{d}{dx}(u \cdot v) = \frac{du}{dx} \cdot v + u \cdot \frac{dv}{dx}.
\]

This rule can be extended to the product of more functions:

\[
(uvw)' = u'vw + uv'w + uvw' \quad \text{or} \quad \frac{d}{dx}(u \cdot v \cdot w) = \frac{du}{dx} \cdot v \cdot w + u \cdot \frac{dv}{dx} \cdot w + u \cdot v \cdot \frac{dw}{dx}.
\]

Example: Find the derivative of \( h(x) = x^3 \cos(x) \).

\[
h'(x) = (x^3)' \cdot \cos x + x^3 \cdot (\cos x)'
\]

\[
h'(x) = 3x^2 \cdot \cos x + x^3 \cdot (-\sin x)
\]

\[
h'(x) = 3x^2 \cos x - x^3 \cdot \sin x
\]
Example: If \( y = (2x + 5)(x^4 + x^2 + 6) \), \( \frac{dy}{dx} = ? \)

\[
\frac{dy}{dx} = 2 \cdot (x^4 + x^2 + 6) + (2x + 5) \cdot (4x^3 + 2x)
\]

\[
\left. \frac{dy}{dx} \right|_{x=1} = 2 \cdot (8) + 7 \cdot (6) = 16 + 42 = 58
\]
Theorem: The Reciprocal Rule

If $f$ is differentiable at $x$ and $f(x) \neq 0$, then so is the reciprocal $\frac{1}{f}$. Moreover,

$$
\frac{d}{dx}\left[ \frac{1}{f(x)} \right] = -\frac{f'(x)}{[f(x)]^2}
$$

Exercise: For $g(x) = \frac{1}{x^2 - x}$, find $g'(2)$.

$$
g'(x) = - \frac{[f'(x)]}{[f(x)]^2} \quad \text{where} \quad f(x) = x^2 - x
$$

$$
g'(x) = - \frac{(2x-1)}{(x^2-x)^2}
$$

$$
g'(2) = - \frac{2 \cdot 2 - 1}{(4 - 2)^2} = - \frac{3}{4}
$$
Theorem: The Quotient Rule

If \( f \) and \( g \) are differentiable at \( x \) and \( g(x) \neq 0 \), then the quotient \( f / g \) is differentiable at \( x \) and

\[
\left( \frac{f}{g} \right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}
\]

This formula may be written as:

\[
\left( \frac{u}{v} \right)' = \frac{u' \cdot v - u \cdot v'}{v^2}
\]

Example: Find the derivative of \( f(x) = \frac{2x}{x^2 + 5} \).

\[
f'(x) = \frac{(2x)' \cdot (x^2 + 5) - (2x) \cdot (x^2 + 5)'}{(x^2 + 5)^2}
\]

\[
f'(x) = \frac{2 \cdot (x^2 + 5) - 2x \cdot 2x}{(x^2 + 5)^2}
\]

\[
f'(x) = \frac{10 - 2x^2}{(x^2 + 5)^2}
\]
Example: Find the derivative of \( f(x) = \frac{\sin x}{5x + 2} \).

\[
f'(x) = \frac{\cos x \cdot (5x + 2) - \sin x \cdot (5)}{(5x + 2)^2}
\]

\[
f'(x) = \frac{(5x + 2) \cdot \cos x - 5 \sin x}{(5x + 2)^2}
\]
Example: Find the slope of the tangent line to the curve \( f(x) = \frac{x^2 + x}{4x + 5} \) at \( x = 1 \).

\[
f'(x) = \frac{(2x+1)(4x+5) - (x^2+x) \cdot (4)}{(4x+5)^2}
\]

\[
f'(1) = \frac{3 \cdot 9 - 2 \cdot 4}{(4 \cdot 1 + 5)^2} = \frac{27 - 8}{81} = \frac{19}{81}
\]
\[
(f + g)' \\
(f - g)' \\
(f \cdot g)' \\
\left( \frac{f}{g} \right)' \\
(f \circ g)' = \text{Composition} \\
(f \circ g)' = \left[ f(g(x)) \right]' \\
= f'(g(x)) \cdot g'(x)
\]
ex: \((f \circ g)'(2) = f'(g(2)) \cdot g'(2)\)

\[g\]

easy!

When chain rule?

\[g(x) = x^5 \quad h(x) = 4x + 1\]

Case-1

\[f(x) = \boxed{(4x + 1)^5}\]

\[f'(x) = ?\]

\[\text{(expression)}^5\]

usual = \[\text{deriv. base of }\]

\[\text{deriv. base of}\]

\[\text{chain rule}\]

\[(ugly)^5 \overset{\text{deriv}}{\Rightarrow} 5 \cdot (ugly)^4 \cdot (ugly)'\]
Theorem: The Chain Rule

If \( g \) is differentiable at \( x \) and \( f \) is differentiable at \( g(x) \), then the composition \( f \circ g \) is differentiable at \( x \). Moreover,

\[
(f \circ g)'(x) = f'(g(x)) \cdot g'(x).
\]

This rule is one of the most important rules of differentiation. It helps us with many complicated functions.

Example: Find the derivative of \( h(x) = (2x+1)^3 \).

\[
h'(x) = 3 \cdot (2x+1)^2 \cdot (2x+1)'
\]

\[
h'(x) = 3 \cdot (2x+1)^2 \cdot 2 \quad = \quad 6 (2x+1)^2
\]
Example: Find the derivative of $h(x) = (x^2 + 5x)^4$.

$$h'(x) = 4 \cdot (x^2 + 5x)^3 \cdot (2x + 5)$$

Example: Find the derivative of $h(x) = (x^3 + x + 1)^{10}$.

$$h'(x) = 10 \cdot (x^2 + x + 1)^9 \cdot (3x^2 + 1)$$

$$h'(0) = 10 \cdot (0 + 1)^9 \cdot (3 \cdot 0 + 1) = 10$$
POPPER #

Question# \[ y = \sin x, \quad \frac{d^3y}{dx^3} = ? \]

a) \( \cos x \)
b) \( \sin x \)
c) \( -\sin x \)
\[ \text{d) } -\cos x \]
e) None

\[ \frac{dy}{dx} = \cos x \quad \Rightarrow \quad \frac{d^2y}{dx^2} = \ldots \]

\[ \Rightarrow \quad \frac{d^3y}{dx^3} = \ldots \]

Question# \[ f(x) = \frac{x}{x+2}, \quad f'(2) = ? \]

a) \( \frac{1}{2} \)
b) \( \frac{1}{4} \)
c) \( \frac{1}{8} \)
\[ \text{d) } \frac{1}{16} \]
e) None

\[ f'(x) = \frac{1(x+2) - x.1}{(x+2)^2} \]

Question# \[ f(x) = x \cos(x), \quad f'(\pi) = ? \]

a) 0 
b) 1 
c) -1 
d) 2 
e) None