If you email me, please mention the course (1432) in the subject line.

Bubble in PS ID and Popper Number very carefully. If you make a bubbling mistake, your scantron can’t be saved in the system. In that case, you will not get credit for the popper even if you turned it in.

Check your CASA account for Quiz due dates. Don’t miss any online quizzes!

Be considerate of others in class. Respect your friends and do not distract anyone during the lecture.

POPPER #

Question # Which of the following is a convergent improper integral?

A) \( \int_{1}^{\infty} \frac{1}{x} \, dx \)  
B) \( \int_{1}^{\infty} \frac{1}{\sqrt{x}} \, dx \)  
C) \( \int_{1}^{4} \frac{1}{x} \, dx \)  
D) \( \int_{1}^{\infty} \frac{1}{x^2} \, dx \)

Question # Which of the following is/are divergent improper integral(s)?

I. \( \int_{1}^{\infty} \frac{1}{x^3} \, dx \)  
II. \( \int_{1}^{\infty} \frac{1}{x^{3/2}} \, dx \)  
III. \( \int_{1}^{2} \frac{1}{x-2} \, dx \)

A) I only  
B) II only  
C) III only  
D) I and II  
E) II and III
Section 7.7 Improper Integrals (continued)

Types of improper integrals:

A. (one or both bounds are infinite)

\[ \int_{1}^{\infty} \frac{dx}{x}, \int_{-\infty}^{1} \frac{3dx}{x^4 + 5} \text{ and } \int_{-\infty}^{\infty} \frac{dx}{x^2 + 1} \text{ are improper because one or both bounds are infinite.} \]

B. (infinite discontinuity at a boundary)

\[ \int_{1}^{5} \frac{dx}{\sqrt{x - 1}} \text{ is improper because } f(x) = \frac{1}{\sqrt{x - 1}} \text{ has an infinite discontinuity at } x = 1. \]

C. (infinite discontinuity in the interior)

\[ \int_{-2}^{2} \frac{dx}{(x + 1)^2} \text{ is improper because } f(x) = \frac{1}{(x + 1)^2} \text{ has an infinite discontinuity at } x = -1, \text{ and } -1 \text{ is between } -2 \text{ and } 2. \]
More examples about the second and third type of improper integral:

Example: Determine if the integral is improper; explain why. Determine if convergent or divergent.

\[ \int_{0}^{4} \frac{e^{1+\sqrt{x}}}{\sqrt{x}} \, dx \]

Example: Determine if the integral is improper; explain why. Determine if convergent or divergent.

\[ \int_{1}^{4} \frac{dx}{x-2} \]
Exercise: \( \int_{0}^{1} (1-x)^{-2/3} \, dx \)

POPPER#

Question # \( \int_{2}^{3} \frac{1}{(x-2)^2} \, dx \)

A) 1  B) 0  C) 1/2  D) ln(2)  E) Divergent

Question # \( \int_{0}^{\infty} e^{-2x} \, dx \)

B) 1  B) 1/2  C) 2  D) -2  E) Divergent
Chapter 8 - Techniques of Integration

Example: $\int x e^{x^2 + 1} dx$

Example: $\int x e^x dx$

Section 8.1 - Integration by Parts

Let’s start with the product rule:

$$\frac{d}{dx}(uv) = u \frac{d}{dx} v + v \frac{d}{dx} u$$

So, the integration by parts formula is:

$$\int u dv = uv - \int v du$$
We use it to “undo” the product rule.

1. **Reduction to integrate**  
   \[ x^n \sin(ax), x^n \cos(ax), x^n e^{ax}, \]  
   \[ \text{polynomial} \cdot \sin(ax), \text{polynomial} \cdot \cos(ax), \text{polynomial} \cdot e^{ax} \]  

2. **Cycling to integrate**  
   \[ \cos(ax) \sin(bx), \cos(ax) e^{bx}, \sin(ax) e^{bx} \]

3. **Change of Form to integrate**  
   \[ \ln(x) f(x), \arctan(x) f(x), \arcsin(x) f(x) \]  
   (where \( f(x) \) has a simple antiderivative)

How do you know what to pick for \( u \) and for \( dv \)?

“Shortcut”: **ILATE**

<table>
<thead>
<tr>
<th>Type of function</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>I – Inverse trigonometric functions</td>
<td>arcsin(x), arctan(x), etc.</td>
</tr>
<tr>
<td>L – Logarithmic functions</td>
<td>ln(x), log₂(x), etc.</td>
</tr>
<tr>
<td>A – Algebraic functions</td>
<td>( x^3, 5x^2, ) etc.</td>
</tr>
<tr>
<td>T – Trigonometric functions</td>
<td>sin(x), tan(x), etc.</td>
</tr>
<tr>
<td>E – Exponential functions</td>
<td>( e^x, 2^x, ) etc.</td>
</tr>
</tbody>
</table>

Functions which are higher on the list are more likely to be selected for \( u \) and functions lower on the list are more likely to be used as \( dv \) (don’t forget to include \( dx \) in this term!).
Example 1: \( \int x e^x \, dx \)

Example 2: \( \int x \sin(x) \, dx \)
Example 3: \( \int x^2 \cos(x) \, dx \)

Example 4: \( \int x^2 \ln(5x) \, dx \)