Math 1432 DAY 14
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If you email me, please mention the course (1432) in the subject line.

Check your CASA account for Quiz due dates. Don’t miss any online quizzes!

Be considerate of others in class. Respect your friends and do not distract anyone during the lecture.

Question# \[ \int \arcsin(x) \, dx = \]

A) \( \arcsin(x)\sqrt{1-x^2} + C \)

B) \( x\arcsin(x) + \frac{1}{2} \sqrt{1-x^2} + C \)

C) \( x\arcsin(x) - \sqrt{1-x^2} + C \)

D) \( x\arcsin(x) + \sqrt{1-x^2} + C \)

E) None of these
Section 8.2 Powers and Products of Trigonometric Functions

Recall the following identities:

\[
\cos^2(x) + \sin^2(x) = 1 \\
1 + \tan^2(x) = \sec^2(x) \\
1 + \cot^2(x) = \csc^2(x)
\]

\[
\cos^2(x) = \frac{1 + \cos(2x)}{2} \\
\sin^2(x) = \frac{1 - \cos(2x)}{2}
\]

\[
\sin(2x) = 2\sin(x)\cos(x)
\]

\[
\cos(2x) = \cos^2(x) - \sin^2(x)
\]

In this section, we will study techniques for evaluating integrals of the form

\[
\int \sin^m x \cos^n x \, dx \quad \text{and} \quad \int \sec^m x \tan^n \, dx
\]

where either \(m\) or \(n\) is a positive integer.

To find antiderivatives for these forms, try to break them into combinations of trigonometric integrals to which you can apply a formula:

\[
\int u^n \, du = \begin{cases} 
\frac{u^{n+1}}{n+1} + C & \text{if } n \neq 1 \\
\ln|u| + C & \text{if } n = -1
\end{cases}
\]
Integrals Involving Powers of Sine and Cosine

1. **If m or n odd:**
   
   a. *m odd:* rewrite $\sin^m x$ as $\sin^{m-1} x \sin x$ ($m$-1 is even so can use identity $\sin^2 x = 1 - \cos^2 x$)
   
   b. *n odd:* rewrite $\cos^m x$ as $\cos^{m-1} x \cos x$ ($n$-1 is even so can use identity $\cos^2 x = 1 - \sin^2 x$)

Example 1: $\int \sin^3 x \, dx$

Example 2: $\int \sin^3 x \cos^2 x \, dx$
Example 3: \[ \int \cos^5 x \, dx \]

Example 4: \[ \int \sin^4 x \cos^3 x \, dx \]
2. If \( m \) and \( n \) even use these identities:

\[
\sin^2 x = \frac{1 - \cos (2x)}{2} \quad \text{and/or} \quad \cos^2 x = \frac{1 + \cos (2x)}{2}.
\]

Example 5: \( \int \cos^2 x \, dx \)

Note:

\[
\int \sin^2 x \, dx = \frac{1}{2} x - \frac{1}{2} \sin x \cos x + C
\]

\[
\int \cos^2 x \, dx = \frac{1}{2} x + \frac{1}{2} \sin x \cos x + C
\]
Recall from Calculus 1:

\[ \int \sec(x) \, dx = \ln|\sec x + \tan x| + C \]

\[ \int \tan(x) \, dx = -\ln|\cos x| + C = \ln|\sec x| + C \]

\[ \int \sec^2(x) \, dx = \tan x + C \]

\[ 1 + \tan^2 x = \sec^2 x \]

\[ \tan^2 x = \sec^2(x) - 1 \]

**Integrals involving Secants and Tangents**

\[ \tan^2 x + 1 = \sec^2 x \]

For \( \int \tan^m x \sec^n x \, dx \)

a. \( n \) even: rewrite \( \tan^m x \sec^n x \) as \( \tan^m x \sec^{n-2} x \sec^2 x \) (then you can use identity \( \sec^2 x = \tan^2 x + 1 \))

b. \( m \) odd: rewrite as \( \tan^{m-1} x \sec^{n-1} x \cdot \sec x \tan x \) \((m-1) \) is even so can use identity \( \tan^2 x = \sec^2 x - 1 \)

c. \( m \) even and \( n \) odd: rewrite \( \tan^m x \) using \( \tan^2 x = \sec^2 x - 1 \)

**NOTE:** We already covered an important integral using IbP:

**Formula:**

\[ \int \sec^3(u) \, du = \frac{1}{2} \sec(u) \tan(u) + \frac{1}{2} \ln|\sec(u) + \tan(u)| + C \]
Example 6: \[ \int \tan^3 x \, dx \]

Example 7: \[ \int \sec^4 x \, dx \]
Example 8: \( \int \sec^4 x \tan^2 x \, dx \)

Example 9: \( \int \sec^5 x \tan x \, dx \)

Other formulas:

1. \( \int \sin^2 x \, dx = \frac{1}{2} x - \frac{1}{2} \sin x \cos x + C \)
2. \( \int \cos^2 x \, dx = \frac{1}{2} x + \frac{1}{2} \sin x \cos x + C \)
3. \( \int \tan^n x \, dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x \, dx \quad n \geq 2 \)
4. \( \int \sec^n x \, dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx \quad n \geq 2 \)
POPPER#

Q# \[ \int \cos x \sin^3 x \, dx \]

a. \[ \frac{1}{2} \cos^2 x + C \]  
   b. \[ \frac{1}{4} \cos^4 x + C \]

c. \[ \frac{1}{4} \sin^4 x + C \]  
   d. \[ -\frac{1}{4} \sin^4 x + C \]

e. none of these