Math 1432

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Visit CASA regularly for announcements and course material!

Read the syllabus posted on the course website.

If you email me, please mention the course (1432) in the subject line.

Respect your friends- do not distract anyone during lectures.

Review Continued:

Example: Given that \( f''(x) = 6x + 4, \ f'(0) = 5, \ f(1) = 2 \), find \( f(2) \).

\[
\begin{align*}
\int f(x) \, dx &= f' \rightarrow f'' \rightarrow f^{(1)} \\

f'(x) &= \int f''(x) \, dx = \int (6x + 4) \, dx = 3x^2 + 4x + C \\

f'(0) &= 5 \quad \Rightarrow \quad 0 + 0 + C = 5 \quad \Rightarrow \quad C = 5 \\

f'(x) &= 3x^2 + 4x + 5 \\

f(x) &= \int f'(x) \, dx = \int (3x^2 + 4x + 5) \, dx \\

&= x^3 + 2x^2 + 5x + D \\

f(1) &= 1 + 2 + 5 + D = 2 \quad \Rightarrow \quad 8 + D = 2 \quad \Rightarrow \quad D = -6 \\

f(2) &= 2^3 + 2 \cdot 2^2 + 5 \cdot 2 + -6 = \boxed{20}
\end{align*}
\]
Recall FTOC!

\[ f(x) = \int_{a}^{x} f(t) \, dt \]

\[ f'(x) = \frac{d}{dx} \left( \int_{a}^{x} f(t) \, dt \right) \]

Example: Given that \( f(x) = \int_{1}^{2x} t \sin(t) \, dt \), find the instantaneous rate of change of \( f(x) \) at \( x = \frac{\pi}{4} \).

\[ f'(x) = \left[ \int_{1}^{2x} t \sin(t) \, dt \right]^{1} \]

\[ = 2x \cdot \sin(2x) \cdot 2 = 4x \cdot \sin(2x) \]

\[ f'(\frac{\pi}{4}) = 4 \cdot \frac{\pi}{4} \cdot \sin\left(2 \cdot \frac{\pi}{4}\right) = \pi \cdot \frac{1}{2} = \frac{\pi}{2} \]
Note: “Exercises” will be left for you as an exercise; the solution for most exercises will be included in the notes but later – after you try them.

EXERCISE: Given that \( f(x) \) is a differentiable function and 
\[
\int_0^x (f(t) + 6t) \, dt = \cos(x),
\]
find \( f''\left(\frac{\pi}{3}\right) \).

**Derivatives of both sides:**
\[
\begin{align*}
f(x) + 6x & = (\cos(x))' = -\sin(x) \\
f'(x) & = -6x - \sin(x) \\
p'(x) & = -6 - \cos(x) \\
p''\left(\frac{\pi}{3}\right) & = -6 - \cos\left(\frac{\pi}{3}\right) = -6 - \frac{1}{2} = \frac{-13}{2}
\end{align*}
\]
Section 7.2 – Average Value of a Function

First Mean Value Theorem for Integrals:

If \( f \) is continuous on \([a, b]\), then there is at least one number \( c \) in \((a, b)\) for which

\[
\int_{a}^{b} f(x) \, dx = f(c)(b-a)
\]

The number \( f(c) \) is called the **average (mean) value** of \( f \) on \([a, b]\).
The area of the region under the graph of \( f \) is equal to the area of the rectangle whose height is the average value.

So….

If \( f \) is integrable on \([a, b]\), then the average value of \( f \) on the interval is:

\[
\text{Average value} = f(c) = \frac{1}{b-a} \int_a^b f(x) \, dx = \frac{\int_a^b f(x) \, dx}{b-a}
\]
Example: Find the average value of the function over the indicated interval.

Find the value(s) of x (the value(s) of c) in the interval for which the function equals its average value:

\[ f(x) = 3x^2 - 2, \quad [0, 2] \]

\[
\text{Avg} = \frac{1}{b-a} \cdot \int_{a}^{b} f(x) \, dx = \frac{1}{2-0} \int_{0}^{2} (3x^2 - 2) \, dx
\]

\[
= \frac{1}{2} \cdot \left[ x^3 - 2x \right]_{0}^{2} = \frac{1}{2} (4 - 0) = 2
\]

Solve \( f(c) = 2 \)

3. \( c^2 - 2 = 2 \) \( \Rightarrow \) \( 3c^2 = 4 \)

\( \Rightarrow c^2 = \frac{4}{3} \)

\( \Rightarrow c = \pm \sqrt{\frac{4}{3}} \)

\( \text{in } (0, 2) \) \( \Rightarrow c = \sqrt{\frac{4}{3}} \)
Example: Find the average value of the function over the given interval:

\[ f(x) = 4x^3 + 6x, \quad [-1, 1] \]

\[
\text{Avg} = \frac{1}{1 - (-1)} \int_{-1}^{1} (4x^3 + 6x) \, dx
\]

\[
= \frac{1}{2} \left[ x^4 + 3x^2 \right]_{-1}^{1} = \frac{1}{2} \left( 1^4 - (-1)^4 \right) = 0
\]
Example: Given that the average value of an even function over \([-2,2]\) is 10, find

\[
\frac{1}{2-(-2)} \int_{-2}^{2} f(x) \, dx = 10
\]

Exercise: An object is in rectilinear motion with acceleration \(a(t) = 6t + 2, \ t \geq 0\).

If the initial velocity is 0, find the average speed of this object over the first 2 seconds.

\[
\begin{align*}
\mathbf{v}(t) &= \int a(t) \, dt = \int (6t+2) \, dt = 3t^2 + 2t + C \\
\text{initial velocity} = 0 \Rightarrow C = 0 \quad \mathbf{v}(t) = 3t^2 + 2t > 0 \\
\text{speed} &= |\mathbf{v}(t)| = 3t^2 + 2t \\
\text{average speed} &= \frac{1}{b-a} \int_{a}^{b} \text{speed} \, dt \\
&= \frac{1}{2-0} \int_{0}^{2} (3t^2 + 2t) \, dt = \frac{1}{2} \left[ t^3 + t^2 \right]_{0}^{2} = \frac{1}{2} \cdot 12 = 6
\end{align*}
\]
Section 7.3  Area Under the Graph of a Nonnegative Function

Fact: If \( y = f(x) \) is nonnegative and integrable over the interval \([a,b]\), then the area under the curve \( y = f(x) \) over \([a,b]\) is:

\[
A = \int_a^b f(x) \, dx.
\]

Example: Find the area under the curve \( f(x) = x^2 - 1 \) from \( x=1 \) to \( x=2 \).

\[
\text{area} = \int_1^2 (x^2 - 1) \, dx
= \left[ \frac{x^3}{3} - x \right]_1^2
= \left( \frac{8}{3} - 2 \right) - \left( \frac{1}{3} - 1 \right)
= \frac{2}{3} + \frac{2}{3} = \frac{4}{3}
\]
What if \( f(x) \) is BELOW the x-axis?

What if it is sometimes above sometime below the x-axis?
Example: Find the area of the region bounded by the graph of \( f(x) = \sin x \) over the interval \([0, 2\pi]\).

\[
\int_0^{2\pi} \sin x \, dx + \int_{\pi}^{2\pi} \sin x \, dx
\]

\[
= 2 \int_0^{\pi} \sin x \, dx = 2 \left[ -\cos x \right]_0^{\pi}
\]

\[
= 2 \left[ -(-1) - (-1) \right] = 2 \left[ 2 \right] = 4
\]

Take practice test 1 and test 1 SOON!
Review Chapter 6 using Calculus 1 links on CASA or on my website.
Homework #1 is posted on CASA.

Check CASA regularly for announcements.