Math 1432

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Purchase Popper scantrons with your section number from UH Bookstore.

Respect your friends. Do not distract anyone during lectures.
Section 7.4 - Volume

Part 2 - Finding the volume of a solid of revolution: DISKS and WASHERS

Recall:

Volume of a disk with base radius \( r \) and thickness \( h \): \( V = \pi r^2 h \)
Volume by DISKS and WASHERS

Volume by Disks for Rotation About the $x$-axis

$$V = \int_a^b A(x) \, dx = \int_a^b \pi [R(x)]^2 \, dx.$$

Volume by Disks for Rotation About the $y$-axis

$$V = \int_c^d A(y) \, dy = \int_c^d \pi [R(y)]^2 \, dy.$$
Disk Method:

**Revolving about the x-axis:** \[ V = \int_a^b \pi [f(x)]^2 \, dx \]

**Revolving about the y-axis:** \[ V = \int_c^d \pi [g(y)]^2 \, dy \]

**Example:** Let \( R \) be the region bounded by the x-axis and the graphs of \( y = \sqrt{x} \) and \( x = 4 \). Sketch and shade the region \( R \). Label points on the x and y-axis.

a) Give the formula for the volume of the solid generated when the region \( R \) is rotated about the x-axis.

\[ V = \int_0^4 \pi \cdot (\sqrt{x})^2 \, dx \]

\[ V = \int_0^4 \pi x \, dx \]

\[ V = \pi \left[ \frac{x^2}{2} \right]_0^4 \]

\[ V = \pi \left[ 8 - 0 \right] \]

\[ V = 8\pi \]

b) Find the volume for the solid in (a).
**Example:** Let R be the region bounded by the y-axis and the graphs of \( y = 2\sqrt{x} \) and \( y = 2 \). Sketch and shade the region R. Label points on the x and y-axis.

a) Give the formula for the volume of the solid generated when the region R is rotated about the y-axis.

\[
V = \pi \int_0^2 (y^2) \, dy
\]

b) Find the volume for the solid in (a).
Example: Consider the region in the first quadrant enclosed by \( y = 4 - x^2 \).

Set up the integral that gives the volume of the solid formed by revolving this region about the x-axis.

\[
V = \int_{0}^{2} \pi \left( 4 - x^2 \right)^2 \, dx
\]
Example: Let R be the region in the first quadrant bounded by the y-axis and the graphs of \( y = x^2 \) and \( y = 9 \). Sketch and shade the region R.

Give the formula for the volume of the solid generated when the region R is rotated about the y-axis. Find the volume for the solid.

\[
V = \int_0^9 \pi (\sqrt{y})^2 \, dy
\]

\[
= \pi \int_0^9 y \, dy
\]

\[
= \pi \left[ \frac{y^2}{2} \right]_0^9
\]

\[
= \frac{81\pi}{2}
\]
**Example:** Rotate the region enclosed by \( y = \sqrt{\sin x}, \ 0 < x < \pi, \) about the x-axis. Determine the volume of the solid formed.

\[
V = \int_0^\pi \pi \cdot \left( \sqrt{\sin x} \right)^2 \, dx
\]

\[
V = \pi \left[ -\cos x \right]_0^\pi = 2\pi
\]

**Exercise:** Let \( R \) be the region bounded by the graph of \( f(x) = \frac{1}{\sqrt{x+1}} \) and the x-axis for \( x \in [0,8] \). Set up the formula that gives the volume of the solid generated by rotating \( R \) about the x-axis.

\[
V = \int_0^8 \pi \cdot \left( \frac{1}{\sqrt{x+1}} \right)^2 \, dx
\]

\[
V = \pi \left[ \ln(x+1) \right]_0^8 = \pi \cdot \ln 9
\]
What if we rotate around a different line?

**Example:** Consider the region enclosed by \( y = \sqrt{x}, x = 1, x = 4 \) and \( y = 1 \). Give the formula for the volume of the solid formed by revolving this region around the line \( y = 1 \).

\[
V = \pi \int_1^4 (\sqrt{x} - 1)^2 \, dx
\]

**Example:** Consider the region enclosed by \( y = x^2, y = 0, x = 3 \). Set up the integral that gives the volume of the solid formed by revolving this region around the line \( x = 3 \).

\[
V = \pi \int_0^9 (3 - \sqrt{y})^2 \, dy
\]
WASHERS

When we apply the same idea (parallel cross sections) to a region that is not containing the axis of revolution, we might get “washers” instead of disks.

Revolving about the x-axis: \[ V = \int_{a}^{b} \pi \left( [f(x)]^2 - [g(x)]^2 \right) dx \]

Revolving about the y-axis: \[ V = \int_{c}^{d} \pi \left( [F(y)]^2 - [G(y)]^2 \right) dy \]