Math 1432  DAY 7

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If you email me, please mention the course (1432) in the subject line.

Check your CASA account for Quiz due dates. Don’t miss any online quizzes!

Be considerate of others in class. Respect your friends, do not distract any one during lectures.

Make sure you bubble in PS ID and Popper number carefully; if you make a mistake, your popper will not be saved in the system and you will not get credit even if you were in class.

POPPER #  O  

Question Let R be the region bounded by \( y = x \) and \( y = \sqrt{x} \). Set up the integral that gives the volume of the solid formed when R is rotated about the x-axis.

A) \( \int_{0}^{1} (x - x^2) \, dx \)

B) \( \int_{0}^{1} \pi (x^2 - x) \, dx \)

C) \( \int_{0}^{1} \pi (x - x^2) \, dx \)

D) \( \int_{0}^{1} \pi (y^2 - y) \, dy \)

E) \( \int_{0}^{1} \pi (y - y^2) \, dy \)
SHELL METHOD

In the **Disk Method**, the rectangle of revolution is perpendicular to the axis of revolution.

In the **Shell Method**, the rectangle of revolution is parallel to the axis of revolution.

Shell Formula for Revolution About a Vertical Line

The volume of the solid generated by revolving the region between the x-axis and the graph of a continuous function $y = f(x) \geq 0$, $L \leq a \leq x \leq b$, about a vertical line $x = L$ is

$$V = \int_a^b 2\pi \left( \frac{\text{shell}}{\text{radius}} \right) \left( \frac{\text{shell}}{\text{height}} \right) dx.$$
> Revolving about the y-axis or a vertical axis: \[ V = \int_a^b 2\pi r(x) h(x) \, dx \]

> Revolving about the x-axis or a horizontal axis: \[ V = \int_c^d 2\pi r(y) h(y) \, dy \]

\[ V = \int 2\pi \cdot r \cdot h \, dx \]
Example: Let $R$ be the region in the first quadrant bounded by $y = 4 - x^2, \ y = 0$. Find the volume of the solid formed by rotating $R$ about the $y$–axis using shell method.

\[ V = \int 2\pi \cdot r \cdot h \cdot dx \]

\[ V = \int_{0}^{2} 2\pi \cdot x \cdot (4 - x^2) \cdot dx \]

\[ V = 2\pi \int_{0}^{2} (4x - x^3) \, dx \]

\[ V = 2\pi \left[ 2x^2 - \frac{x^4}{4} \right]_{0}^{2} \]

\[ V = 2\pi \left[ 8 - 4 \right] = 8\pi \]
**Example:** Let \( R \) be the region in the first quadrant bounded by \( y = 4 - x^2, \ y = 0 \). Find the volume of the solid formed by rotating \( R \) about the \( x \)-axis using **shell method**.

\[ V = \int_{0}^{4} 2\pi \cdot r \cdot h \cdot dy \]

\[ V = \int_{0}^{4} 2\pi \cdot y \cdot \sqrt{4 - y} \cdot dy \]

\( \text{Area} = 4 - x^2 \)

\( x^2 = 4 - y \Rightarrow x = \sqrt{4 - y} \)
**Example:** Let R be the region bounded by the graph of \( f(x) = -x^2 + 4x \) and \( g(x) = x^2 \). Set up the integral that gives the volume of the solid generated by rotating R about

a) the y-axis.

Shell

\[ r = x \]

\[ V = \int_0^2 2\pi x \cdot (-x^2 + 4x - x^2) \, dx \]

b) the x-axis.

Disk/washer

\[ V = \pi \int_0^2 (R^2 - r^2) \, dx \]

\[ V = \pi \int_0^2 (-x^2 + 4x^2 - x^2) \, dx \]
Example: Let R be the region bounded by $y = x^2 + 4$, $y = -x + 6$, $x = 0$.

Set up the integral that gives the volume of the solid formed by rotating R about the y-axis.

\[
V = \int_0^6 2\pi \cdot r \cdot h \, dx
\]

where $r = x$, $h = y_{top} - y_{bottom}$, and the integral is taken from $x = 0$ to $x = 6$. The integral for the volume is:

\[
V = \int_0^6 2\pi x \left( (-x + 6) - (x^2 + 4) \right) \, dx
\]
**Exercise:** Let $R$ be the region bounded by $y = \sqrt{x}, y = x^2, 0 \leq x \leq 1$. Set up the integral that gives the volume of the solid formed when $R$ is revolved about the line $x = -2$.

![Diagram](image)

**Exercise:** Let $R$ be the region bounded by the graph of $f(x) = 9 - x^2$ and $g(x) = 2x^2$. Set up the integrals that give the volume of the solid generated by rotating $R$ about

- a) the $x$-axis.
- b) the $y$-axis.
WORKSHEET --- VOLUME

Solve these exercises after class! These are pretty typical problems. Solutions will not be posted; you can bring your questions to me or to your lab TA.

Exercise: Let $\Omega$ be the region bounded by $y = \sqrt{x}$ and $y = x^2$, from $x = 0$ to $x = 1$.

   a. Find the volume of the solid formed by rotating $\Omega$ around the x-axis.

   b. Find the volume of the solid formed by rotating $\Omega$ around the y-axis.

Exercise: Let $\Omega$ be the region bounded by $y = x^3$ and $y = 8x$, from $x = 0$ to $x = 2$.

   a. Find the volume of the solid formed by rotating $\Omega$ around the x-axis.

   b. Find the volume of the solid formed by rotating $\Omega$ around the y-axis.
Solve these exercises after class! These are pretty typical problems.

Exercises:

1. The region bounded by $y = x^3$, $x = 1$ and the x-axis is rotated about the x-axis. Find the volume of the solid formed.

2. The region bounded by $y = \sqrt{\cos x}$, $x = 0$, $x = \pi / 2$ and the x-axis is rotated about the x-axis. Find the volume of the solid formed.

3. The region bounded by $y = x^3$, $y = 8$ and the y-axis is rotated about the y-axis. Find the volume of the solid formed.

4. The region bounded by $y = x^2$, $x = 1$ and the x-axis is rotated about the x-axis. Find the volume of the solid formed.

5. The region bounded by $y = x^2$, $y = 1$ and the y-axis is rotated about the y-axis. Find the volume of the solid formed.
Arc Length

How do we find the arc length?

If the curve is traced by \( y = f(x) \) for \( a \leq x \leq b \), then

\[
L = \int_{a}^{b} \sqrt{1 + \left[ f'(x) \right]^2} \, dx.
\]

If the curve is traced by \( x = g(y) \) for \( c \leq y \leq d \), then

\[
L = \int_{c}^{d} \sqrt{1 + \left[ g'(y) \right]^2} \, dy.
\]
Example: Give a formula for the length of the curve given by \( f(x) = 4 - x^2 \) for \(-1 \leq x \leq 2\).

\[
L = \int_a^b \sqrt{1 + [f'(x)]^2} \, dx = \int_{-1}^2 \sqrt{1 + (-2x)^2} \, dx
\]

\[
= \int_{-1}^2 \sqrt{1 + 4x^2} \, dx
\]

Example: Find the length of the curve traced by \( x = \frac{2}{3} (y-1)^{3/2} \) for \( 1 \leq y \leq 4 \).

\[
L = \int_c^d \sqrt{1 + [g'(y)]^2} \, dy
\]

\[
\frac{dx}{dy} = \frac{2}{y} \cdot \frac{3}{2} (y-1)^{1/2}
\]

\[
= \sqrt{y-1}
\]

\[
L = \int_1^4 \sqrt{1 + (\frac{dy}{dy}-1)^2} \, dy = \int_1^4 \sqrt{1 + y-1} \, dy = \int_1^4 \sqrt{y} \, dy = \left[ \frac{y^{3/2}}{3/2} \right]_1^4 = \frac{2}{3} \left( 4^{3/2} - 1 \right)
\]

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These exercises are similar to some of your online quiz questions. Solutions will be posted in the completed notes.

Exercise: Find the length of the curve given by \( f(x) = \frac{1}{3} x^{3/2} - \frac{1}{\sqrt{x}} \) from \( x = 1 \) to \( x = 9 \).

\[
L = \int_{1}^{9} \sqrt{1 + \left( \frac{1}{3} \cdot \frac{3}{2} x^{1/2} - \frac{1}{2x^{1/2}} \right)^2} \, dx
\]

Exercise: Find the length of the curve given by \( f(x) = \frac{1}{4} x^2 - \frac{1}{2} \ln x \) from \( x = 1 \) to \( x = 2 \).

\[
L = \int_{1}^{2} \sqrt{1 + \left( \frac{1}{2x} - \frac{1}{4x^2} \right)^2} \, dx
\]

\[
L = \int_{1}^{2} \sqrt{1 + \frac{x^2}{4x^2} - 2 \cdot \frac{x}{2x} \cdot \frac{1}{4x^2}} \, dx
\]

\[
L = \int_{1}^{2} \sqrt{1 + \frac{x^2}{4x^2} + \frac{1}{4x^2}} \, dx
\]

\[
L = \int_{1}^{2} \sqrt{\frac{2x^2 + x^4 + 1}{4x^2}} \, dx
\]

Note: \( 2x^2 + x^4 + 1 = (x^2 + 1)^2 \) when expanded!
\[ L = \sqrt{\int_1^2 \frac{x^4 + 2x^2 + 1}{4x^2} \, dx} \]

\[ = \int_1^2 \frac{(x^2 + 1)^2}{4x^2} \, dx \]

\[ = \int_1^2 \frac{x^2 + 1}{2x} \, dx = \int_1^2 \frac{x^2}{2x} + \frac{1}{2x} \, dx \]

\[ = \int_1^2 \frac{1}{2}x + \frac{1}{2x} \, dx = \left[ \frac{1}{2} \cdot \frac{x^2}{2} + \frac{1}{2} \cdot \ln |x| \right]_1^2 \]

\[ = \left( \frac{1}{2} \cdot \frac{4}{2} + \frac{1}{2} \cdot \ln(2) \right) - \left( \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \ln(1) \right) \]

\[ = 1 + \frac{1}{2} \cdot \ln(2) - \frac{1}{4} \]

\[ = \frac{3}{4} + \frac{1}{2} \cdot \ln(2) \]
Write an integral that gives the arclength of the curve $f(x) = x^2$ over the interval [0,2].

A) $\int_{0}^{2} \sqrt{1 + x^2} \, dx$

B) $\int_{0}^{2} \sqrt{1 + x^4} \, dx$

C) $\int_{0}^{2} \sqrt{1 - 4x^2} \, dx$

D) $\int_{0}^{2} \sqrt{1 + 4x^2} \, dx$

E) $\int_{0}^{2} \sqrt{1 - 4x^2} \, dx$

Red on Friday