Math 1432
DAY 8
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If you email me, please mention the course (1432) in the subject line.

Bubble in PS ID and Popper Number very carefully. If you make a bubbling mistake, your scantron can’t be saved in the system. In that case, you will not get credit for the popper even if you turned it in.

Check your CASA account for Quiz due dates. Don’t miss any online quizzes!

Be considerate of others in class. Respect your friends, do not distract any one during lectures.
Section 7.5 Continued– SURFACE AREA

How do you find the surface area of a solid of revolution?
Fact: Let $f$ be a positive, differentiable function with a continuous derivative defined on an interval $[a,b]$. The area of the surface $S$ obtained by revolving $f$ around the x-axis is given by:

$$A(S) = \int_a^b 2\pi f(x) \sqrt{1 + [f'(x)]^2} \, dx.$$ 

Fact: Let $F$ be a positive, differentiable function with a continuous derivative defined on an interval $[c,d]$. The area of the surface $S$ obtained by revolving $F$ around the y-axis is given by:

$$A(S) = \int_c^d 2\pi F(y) \sqrt{1 + [F'(y)]^2} \, dy.$$ 

Example: Let $R$ be the region bounded by the graph of $f(x) = 2x^3$ and the x-axis for $x \in [0,2]$. Set up the integral that gives the surface area of the solid generated when $R$ is rotated about the x-axis.

$$A(S) = \int_a^b 2\pi f(x) \sqrt{1 + [f'(x)]^2} \, dx$$

Exercise: Let $R$ be the region bounded by the graph of $f(x) = \sqrt{4-x^2}$ and the x-axis for $x \in [-2,2]$. Set up the integral that gives the surface area of the solid generated when $R$ is rotated about the x-axis.
Section 7.5 Continued – Another application of integrals:

Finding the Centroid

Where is the centroid of this rectangle if it is made out of homogenous material?

How do we find the centroid (or geometric center) of a region bounded by a curve?

**Fact:** Let \( f \) be a positive, continuous function defined on an interval \([a,b]\). Let \( A \) be the area of the region \( R \) bounded by \( f \) and the x-axis. The centroid \((\bar{x}, \bar{y})\) is given by:

\[
\bar{x} = \frac{\int_{a}^{b} xf(x) dx}{A} \quad \text{and} \quad \bar{y} = \frac{1}{2} \int_{a}^{b} \left[ f(x) \right]^2 dx
\]
Fact: Let $R$ be the region bounded by two continuous functions $f$ and $g$ over the interval $[a,b]$. Let $A$ be the area of the region $R$. The centroid $(\bar{x}, \bar{y})$ is given by:

$$
\bar{x} = \frac{\int_a^b x[f(x) - g(x)] \, dx}{A} \quad \text{and} \quad \bar{y} = \frac{\frac{1}{2} \int_a^b [f(x)]^2 - [g(x)]^2 \, dx}{A}.
$$

For ease of notation, we may use:

$$
\bar{x}A = \int_a^b x[f(x) - g(x)] \, dx \quad \text{and} \quad \bar{y}A = \frac{1}{2} \int_a^b [f(x)]^2 - [g(x)]^2 \, dx.
$$
Example: Let $R$ be the region in the first quadrant bounded by the graph of $f(x) = x^3$ and $g(x) = x$. Find the centroid.

Step 1: Find the area of the region

$$A = \int_{0}^{1} (x - x^3) dx = \left[ \frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 = \frac{1}{4}.$$ 

Step 2: Use formulas to find the centroid:

$$\bar{x}A = \int_{a}^{b} x[f(x) - g(x)] dx =$$

$$\bar{y}A = \frac{1}{2} \int_{a}^{b} [f(x)]^2 - [g(x)]^2 dx$$
Fact: If a region has a line of symmetry, then the centroid lies on that line.

![Diagram of a region with a line of symmetry and its centroid.]

Exercise: Let R be the region bounded by the graph of \( f(x) = x^2 \) and \( y = 4 \). Find the centroid.

![Graph of the region R with the bounding curve and the line y=4.]

Step 1: Find the area of the region

\[
A = \int_{-2}^{2} (4-x^2) \, dx = \left[ 4x - \frac{x^3}{3} \right]_{-2}^{2} = \frac{32}{3},
\]

Step 2: Apply the centroid formulas. (If there is symmetry and if you can guess one of the coordinates, you can use a shortcut.)
Theorem 7.5.1: Pappus’s Theorem on Volumes

- Body of revolution is generated by rotating a plane area about a fixed axis.

- Volume of a body of revolution is equal to the generating area times the distance traveled by the centroid through the rotation.

\[ V = 2\pi y \cdot A \]
**Theorem:** Suppose a solid is generated by revolving a region $R$ about any axis such that $R$ does not cross the axis of rotation. Then, the volume of the solid formed is given by:

$$V = 2\pi \overline{R} A.$$ 

Here, $\overline{R}$ is the distance from the centroid of $R$ to the axis of rotation, and $A$ is the area of the region.

Area of circle = $\pi r^2$

Distance center of circle travels = $2\pi R$

Volume the circle sweeps out = $2\pi R \cdot \pi r^2 = 2\pi^2 R r^2$
Example: Let R be the region in the first quadrant bounded by the graph of \( f(x) = x^3 \) and \( g(x) = x \). The centroid of this region is \( C \left( \frac{8}{15}, \frac{8}{21} \right) \).

Find the volume of the solid formed when this region is revolved about

a) the x-axis.

b) the y-axis.
**Popper #**

R be the region given below, with Area = 12, and centroid (5,2).

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**Question#** If R is revolved about the x-axis, what is the volume of the solid formed?

a) 60pi  
b) 24pi  
c) 120pi  
d) 48 pi  
e) None

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**Question#** If R is revolved about the y-axis, what is the volume of the solid formed?

a) 60pi  
b) 24pi  
c) 120pi  
d) 48 pi  
e) None
Section 7.6 – Differential Equations and Exponential Growth/Decay

What is a differential equation?

An equation that relates an unknown function to one or more of its derivatives is called a **differential equation**.

For example:

\[
\frac{dy}{dt} = 5y
\]

Or

\[
y' = y + 1
\]

are examples or “first order” differential equations. We’ll study differential equations in more depth later.

**HOMEWORK**: Watch the video “Differential Equations and Slope Fields” – the link is under Section 7.6 in your TEXT BOOK.

A first order differential equation is said to be separable if it can be written as:

\[
p(x) + q(y)y' = 0.
\]
How do we solve a separable differential equation?

Example: Find the particular solution for:

\[ y' = 4y \quad \text{and} \quad y(0) = 6. \quad (\text{Suppose } y > 0) \]

Example: Find the general solution for:

\[ y' = -\frac{x}{y}. \quad (\text{Suppose } y > 0) \]