Math 2413 FINAL EXAM

Where: CASA Testing Center, by reservation.

Register as soon as possible. Check your confirmation email for the location.

Double check your date and time. Students are responsible for scheduling their exams before the testing window. Once the exam begins, CASA does not guarantee having available seats. Don't miss your test.

Final is mandatory for all students; no exemptions in this class.

Approx. 22-25 Problems (Some MC, some FR – Details TBA)

Time: 110 minutes

Topic covered: Everything!

Practice Final: 5% is added to the final grade

Teacher Evaluations: +2 points on the final.

The percentage on the final (<u>without any extra credit</u>) will be used to replace one missed test OR the lowest test (if it is better). If it is not better than any of your tests, it will only count as the final exam.

Go over the class notes, work on past HW assignments and lab quizzes. Make sure you work on past reviews for tests 1,2 and 3.

We will solve some of these problems in class – the rest are exercises for you.

This review sheet is not a complete list of what you need to know. There may be questions on the final that are from topics not included in this sheet. Make sure you take the practice final (several times if necessary). This sheet should not be your only source while studying for the final.

GO OVER THE REVIEW SHEETS FOR TESTS 1, 2, 3.

1. Find the derivative of: $f(x) = \frac{4x+1}{5x-2}$.

Know the definition of the derivative.

$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$, provided the limit exists.

2. Given: $f(x)=5x^2-4\sqrt{x}$; which of the following would give the first step in finding the derivative using the definition?

A) $\lim_{x\to 0}\left(\frac{\left(5(x+h)^2-4\sqrt{x+h}\right)-\left(5x^2-4\sqrt{x}\right)}{x}\right)$

B) $\lim_{h\to 0}\left(\frac{\left(5(x+h)^2-4\sqrt{x+h}\right)-\left(5x^2-4\sqrt{x}\right)}{h}\right)$

C) $\lim_{x\to 0}\left(\frac{\left(5(x+h)^2-4\sqrt{x+h}\right)-\left(5x^2-4\sqrt{x}\right)}{h}\right)$

D) $\lim_{h\to 0}\left(\frac{\left(5(x+h)^2-4\sqrt{x+h}\right)+\left(5x^2-4\sqrt{x}\right)}{h}\right)$

E) $\lim_{h\to 0}\left(\frac{\left(5(x+h)^2-4\sqrt{x+h}\right)}{h}\right)$

3. Find the following limits (if they exist):

a. $\lim_{x\to 144} \dfrac{x-144}{\sqrt{x}-12}$

b. $\lim_{x\to 0} \dfrac{5-\dfrac{5}{x^2}}{8+\dfrac{8}{x^2}}$

INCREASING/DECREASING

4. Given $f(x) = \frac{1}{3}x^3 - 6x^2 + 2$; find the intervals over which the function is decreasing.

5. Given f(x) is a polynomial and its first derivative is: $f'(x) = 10(x+2)(x-1)(x-5)$; find the intervals over which the function f(x) is increasing.

CONCAVITY

6. Given $f(x) = \frac{1}{4}x^4 - x^3 + 2$; find the intervals over which the function is concave down.

7. Given f(x) is a polynomial and its second derivative is: $f''(x) = -10(x+4)(x-3)(x-6)$; find the intervals over which the function f(x) is concave up.

8. Compute if possible: $\lim_{x\to\infty} \frac{5x^2+2x-1}{e^{3x}}$

See Review for Test 3 for more L'Hospital's rule questions!

Local and Absolute Extrema:

9. Given $f(x) = x^3 - 3x^2 + 1$; find any local min/max points.

10. Given f(x) is a polynomial with $f(2) = 5, f'(2) = 0, f''(2) = 4;$ classify the point x=2 for this function:
    a) Local min
    b) Local max
    c) Inflection point
    d) Absolute max
    e) Vertical cusp

11. Given $f(x) = x^2 - 2x + 5$ over the interval [-1,2] ; find the **absolute maximum value.**

12. Given f(x) is a polynomial with $f(2)=5,\ f'(2)=11$; write the equation of the tangent line to f(x) at x=2.

13. Find the slope of the tangent line to the curve $x^3+y^2+2xy=40$ at the point (2,4).

14. Find the derivatives:

(a) $f(x) = 5x^3 \cos(2x)$

(b) $f(x) = 4\sin^3(5x^2)$

(c) $f(x) = 2\tan\left(e^{3x}\right)$

(d) $f(x) = 10\cos(3x) + 2\sin\left(4e^{5x}\right)$

(e) $f(x) = 6\cos^5(2x)$

15. Find $\frac{dy}{dx}$

(a) $y = 5\arcsin(2x) + 4\arctan(3x)$

(b) $y = \sec(8x) + 6\tan(5x^2)$

(c) $y = e^{5x} + 7^{4x} + 10^{2x+1}$

(d) $y = 10x^2 \sin(5x)$

(e) $y = \sqrt{2 + \cos(3x)}$

(f) $y = \sqrt{x^2 + 5x + 1}$

16. Find the derivatives of the following:

(a) $f(x) = 5\ln(2x+1) + \ln\left(x^2+1\right)$

(b) $f(x) = 2\log(5x+1) + \ln\left(\sin(x)\right)$

Exercise: If $f(x) = 5x^3 + 2x + 1$ is an invertible function, find the equation of the tangent line to $f^{-1}(x)$ at the point where $x = 8$.

(EXERCISE) Consider the function $f(x) = 3x^4 - 20x^3 + 42x^2 - 36x$.

a. Find the critical numbers of f . Hint: $f'(x) = 12(x-3)(x-1)^2$

b. Give the interval(s) of increase and decrease of f.

c. Give the value(s) of x at which f has either a local minimum or a local maximum.

d. Give the interval(s) where the graph of f is concave up.

e. Give the values of x where the graph of f has inflection.

WORD PROBLEMS – Rate of change (3.1) and Optimization (5.3). Go over the examples we covered in class and on review sheets! I included a few examples here but you need to practice the different types as well.

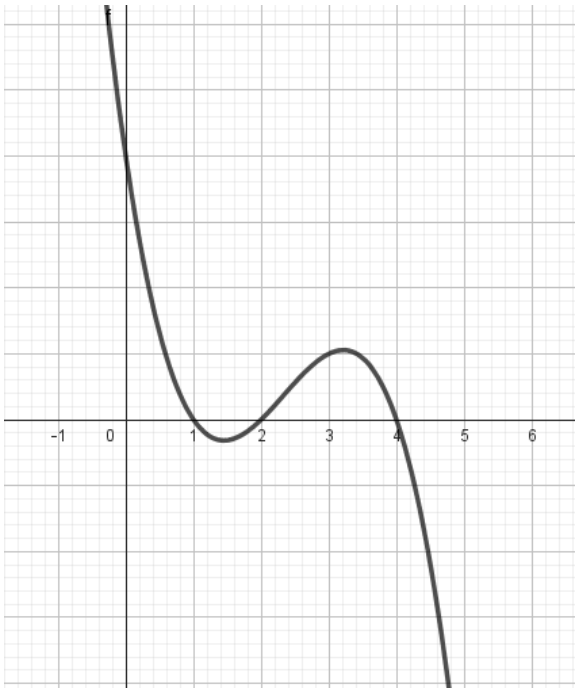
17. The length of a rectangle is increasing at a rate of 5 cm/sec and the width is decreasing at a rate of 2 cm/sec. At what rate is the area changing when length is 12cm and width is 4cm?

Exercise: The base of a triangle is increasing at a rate of 2in/min. If the altitude is always twice the base, how fast is the area changing when the base is 20 inches?

18. Sam is building 5 adjacent rectangular gardens of equal dimensions and area using 1000 feet of fencing material. The goal is to get largest possible area. Write the objective function.

CHAPTER 6

19. The graph of f(x) is given below. Given: the area under f(x) from x=0 to x=1 is 7/3, from x=1 to x=2 is 2/3; the area from x=2 to x=4 is 11/3, compute: $\int_0^4 f(x)dx$.

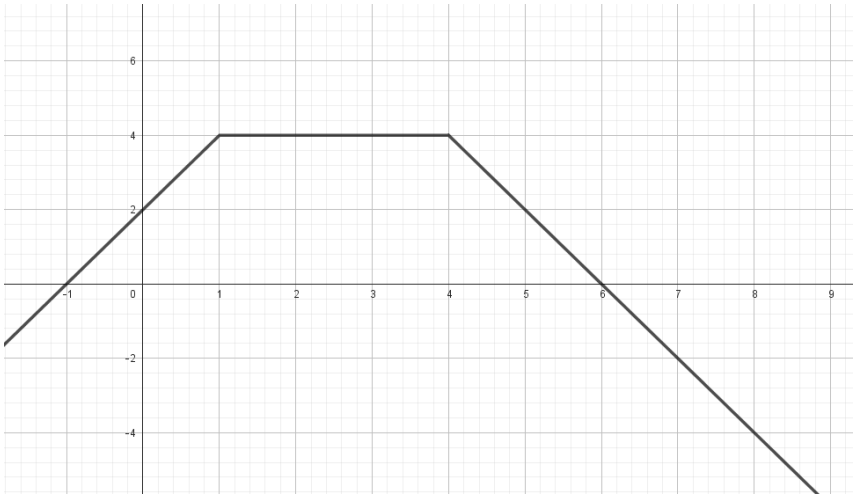


20. Given: $\int_0^{11} f(x)dx = 6, \int_0^5 f(x)dx = 10, \int_4^{11} f(x)dx = 18$; compute: $\int_5^4 f(x)dx$

21. Evaluate:

$$\frac{d}{dx}\left[\int_{-4}^{x^3}\frac{2}{5t+1}\right]dt$$

22. Given the graph of f(x) below, compute: $\int_{2}^{8} f(x)dx$



23. Evaluate:

$$\int_0^1 (x^3 + 2\sqrt{x} + 6)dx$$

24. Rewrite using proper u-substitution: $\int_0^1 6x\sin\left(x^2+1\right)dx$

Choices:

A) $\int_0^1 3\sin(u)du$

B) $\int_1^2 3u\sin(u)du$

C) $\int_1^2 3\sin(u)du$

D) $\frac{1}{3}\int_1^2 \sin(u)du$

E) $\int_1^2 3\sin(u)dx$

F) None of the above

25. Calculate:

$$\int \left(x^3 + \sqrt{x} + 2 \right) dx$$

$$\int \left(e^{4x} + 5^x + \frac{1}{x} \right) dx$$

$$\int 5\sin(2x) + 4\cos(3x)\ dx$$

$$\int 6\sin(5x) + 4\sec^2(x)\ dx$$

$$\int \frac{5}{1+x^2}\ dx$$

$$\int \frac{4}{\sqrt{1-x^2}}\ dx$$

26. Calculate:

$$\int \frac{x}{9+x^2}\,dx$$

$$\int x\sqrt{x^2+2}\;dx$$

$$\int x\left(x^2+1\right)^4\,dx$$

27. Calculate:

a) $\int xe^{x^2}\ dx$

b) $\int \sqrt{5x+1}\ dx$

c) $\int x\cos\left(5x^2\right)\ dx$

28. Compute: $\int_0^2 \frac{5x}{3x^2+2}\,dx = ?$

Take and retake the practice final.

GOOD LUCK ON YOUR FINAL!

Important Exercise: Compute: $\int_0^1 \frac{10x}{6x^2+9}\,dx = ?$

Choices: A) $\frac{5}{6}\left(\ln(15)-\ln(9)\right)$

B) $\frac{5}{6}\left(\ln(15)\right)$

C) $\frac{5}{12}\left(\ln(15)-\ln(9)\right)$

D) $\frac{5}{12}\left(\ln(5)-\ln(3)\right)$

E) None of the above.

Exercise: Rewrite using proper u-substitution: $\int_0^2 5x\sqrt{x^2+5}dx$.

Choices:

A) $5\int_0^2 \sqrt{u}du$

B) $\frac{5}{2}\int_5^9 \sqrt{u}du$

C) $\frac{5}{2}\int_5^9 \sqrt{u}dx$

D) $\frac{1}{2}\int_5^9 \sqrt{u}du$

E) $10\int_5^9 u\sqrt{u}du$

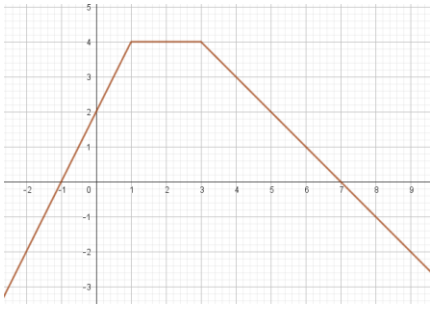
F) None of the above

Exercise: Given: $\int_1^8 f(x)\,dx = 10;\quad \int_2^4 f(x)\,dx = 3;\quad \int_4^8 f(x)\,dx = 4$. Find $\int_2^1 f(x)\,dx = ?$

EXERCISE: The graph of a function f(x) is given below. Compute:

$$\int_0^4 f(x)dx = ?$$

$$\int_5^8 f(x)dx = ?$$



Exercise: Given the following graph of f(x). Determine: $\int_0^5 f(x)dx$