*EHW1 (3.1 & 3.2) → EMCF tab on CASA. Due on Tuesday (2/28)

Math 2413 In-class Review for Test 1

Schedule your exam using "schedule exams" tab on CASA. Students are responsible for scheduling their exams before the testing window begins, while seats are available. Once testing window begins, CASA may not have any available seats. Check your confirmation email to make sure you scheduled it, and to see the exact location.

Plan to be at the testing center early. If this is your first time testing there, they will need a finger print registration. If you are too late, they may not let you in.

When is the test? By reservation during the testing window.

What is covered? Chapter1, Chapter 2.

Number of questions: Approx. 20.

Test 2 ← MC

Test 2 FR

18 multiple choice questions (Total: 76 points)

2 free response questions (Total: 24 points)

The multiple choice part will be graded automatically. The grade you see after your test is for the MC part only – out of ?? points. FR part will be graded later. The graded answer sheet will be returned to you in your labs.

Time allowed: 75 minutes

What to bring? Picture ID and writing utensils. You can ask for scratch paper – you can not bring your own. Do request scratch paper to use for MC questions; work on scratch paper will not be graded. Free response questions must be answered on the ANSWER SHEET that will be provided to you. **The answer sheet has a cover page; it will ask about your instructor's name, and lab info** (lab time and location will be needed so that you can get the answer sheet back after grading).

No notecards, no formula sheets, no calculators, no books. If you bring a cheat sheet/note card to the test, your test will be invalidated and you will be reported to the department for violating UH Academic honesty policies.

How to Study:

- Understand the material we covered in class.
- Go over past HW problems (check the keys!).
- Go over past lab quizzes.
- Work on the MAIN REVIEW sheet posted on CASA.
- Take practice test 1; and retake it. 5% of your best score will be added to your test grade.
- Solve all problems on this review sheet.
- Attend lab and ask your questions. Attend tutoring to get help.

Practice makes perfect!

Review Problems

1. Evaluate the following limits if they do exist.

$$\lim_{x\to 1}\left(\frac{x^2+2x}{x^3-4x+1}\right)$$

$$\lim_{x\to 2}\left(\frac{x^2-4}{x^3-2x^2}\right)$$

$$\lim_{x\to 4}\left(\frac{5x-20}{x^2-16}\right)$$

$$\lim_{x\to 16}\left(\frac{5\sqrt{x}-20}{x-16}\right)$$

$$\lim_{x\to\infty}\left(\frac{x^2-3x+2}{x^3-4x}\right)$$

$$\lim_{x\to\infty}\left(\frac{5x^3-3x+2}{x^3-4x}\right)$$

→ Fact: $\lim_{x\to 0} \frac{\sin(ax)}{bx} = \frac{a}{b}$

2. Evaluate the following limits if they do exist.

$\lim_{x\to 0} \frac{\sin^2(6x)}{7x^2}$ $= \lim_{x\to 0} \underbrace{\frac{\sin(6x)}{7x}} \cdot \underbrace{\frac{\sin(6x)}{x}} = \frac{6}{7}\cdot\frac{6}{1}$ $= \frac{36}{7}$

0/0

$\lim_{x\to 0} \frac{\sin(4x)}{\cos(9x)}$ $= \frac{0}{1} = \boxed{0}$

$\lim_{x\to 0} \frac{\sin(2x)\sin(5x)}{x^2}$ $= \lim_{x\to 0} \underbrace{\frac{\sin(2x)}{x}} \cdot \underbrace{\frac{\sin(5x)}{x}} = \frac{2}{1}\cdot\frac{5}{1}$ $= \boxed{10}$

$\lim_{x\to 0}\left(\frac{\sin(2x)\cos^2(5x)}{3x}\right)$

0/0

$= \lim_{x\to 0} \underbrace{\frac{\sin(2x)}{3x}} \cdot \cos^2(5x)$

$= \lim_{x\to 0} \frac{\sin(2x)}{3x} \cdot \lim_{x\to 0} \cos^2(5x)$ ← plug in

$= \frac{2}{3}\cdot 1 = \boxed{\frac{2}{3}}$

3. $f(x)=\begin{cases} x^3+2, & x<1 \\ 5x, & 1\le x<4 \\ x^2-x, & 4<x \end{cases}$

RHL ≠ LHL

$\lim_{x\to 4^+} f(x)=?$ 16-4 $\lim_{x\to 4^-} f(x)=?$ 5.4 $\lim_{x\to 4} f(x)=?$ DNE

x>4 = 12 x<4 = 20

$\lim_{x\to 1^+} f(x)=?$ 5.1 $\lim_{x\to 1^-} f(x)=?$ 1^3+2 $\lim_{x\to 1} f(x)=?$

Note: Must know how to answer similar questions given a graph.

Piecewise fns.

4. Find the points of discontinuity (if any). Classify each as removable, jump or infinite.

$$f(x)=\begin{cases} x^3-3, & x<-1 \\ \frac{4}{x}, & -1<x<4 \\ x^2-x, & 4\le x \end{cases}$$

Points to check

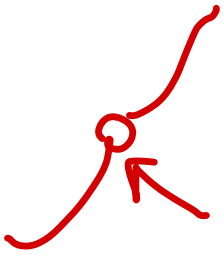
$x=-1$, $x=4$

$x=0$ ← mink discty at $x=0$.

$x=-1$: RHL $= \frac{4}{-1} = -4$

LHL $= (-1)^3-3 = -4$

$f(-1)$: undefined.



Removable discty at $x=-1$.

$x=4$ RHL $= 4^2-4 = 12$ → lim: DNE

LHL $= \frac{4}{4} = 1$ $x\neq 4$

$f(4) = 4^2-4 = 12$

jump discty at $x=4$.

Rational Fns.

5. Find the points of discontinuity (if any). Classify each as removable, jump or infinite.

(a) $f(x)=\dfrac{x^2-4x-5}{x^2-1}$ $= \dfrac{(x-5)(x+1)}{(x-1)\cdot(x+1)} = \dfrac{x-5}{x-1}$

Hole at $x=-1$ ↓ Removable discty at $x=-1$

VA: $x=1$ ↓ infinite discty at $x=1$.

(b) $f(x)=\dfrac{x^2-4x+4}{x^3-4x^2}$ $= \dfrac{(x-2)^2}{x^2(x-4)}$

$x=0$, $x=4$ ↓ infinite discty.

6. Determine whether IVT can be used to conclude that the function given below has a root inside the given interval.

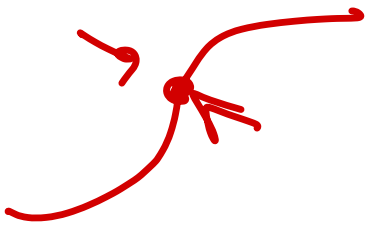
(a) $f(x) = x^3 - x^2 - 1$ over the interval $[0,4]$.

(b) $f(x) = x^3 - x^2 + 1$ over the interval $[0,4]$.

(c) $f(x) = \dfrac{2x}{x^2 - 1}$ over the interval $[0,4]$.

7. Find A and B so that $f(x)$ is continuous:

$$f(x)=\begin{cases} x^2-1 & x<-1 \\ Ax-4 & x=-1 \\ Bx^2+3 & x>-1 \end{cases}$$



Need: $f(-1) = RHL = LHL$ ($x>-1$ for RHL, $x<-1$ for LHL)

$A(-1)-4 = B(-1)^2+3 = (-1)^2-1=0$

$B+3=0 \Rightarrow B=-3$

$-A-4=0 \Rightarrow -A=4 \Rightarrow A=-4$

8. Find A and B so that $f(x)$ is differentiable everywhere:

$$f(x)=\begin{cases} Ax^2-B & x<1 \\ x+2 & x\geq 1 \end{cases}$$

$2A\cdot x$

cts? Need: $f(1) = RHL = LHL$

$3 = 3 = A-B$

$3=\frac{1}{2}-B$

$B=-\frac{5}{2}$

diff? RH deriv. = LH. deriv.

$1 = 2A\cdot 1 \Rightarrow A=\frac{1}{2}$

9. Find the derivative of the following functions using the definition.

$f(x)=\dfrac{2}{x-4}$

Set up the definition: $\lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0} \frac{\frac{2}{(x+h)-4} - \frac{2}{x-4}}{h}$$

$= \ldots$

10. Find the derivatives of the following functions.

(a) $f(x) = x^3 + \frac{1}{x^2} - \sqrt{x} - 5$; find: $f'(x) = ?$

$f'(x) = 3x^2 + (-2)\cdot x^{-3} - \frac{1}{2\sqrt{x}}$

(b) Find: $\frac{d}{dx}\left(\cot(9x) + \tan(4x)\right) = ?$ $\quad -\csc^2(9x)\cdot 9 + \sec^2(4x)\cdot 4$

(b) Find: $\frac{d}{dx}\left(\sec(2x) + \csc(5x)\right) = ?$ $\quad 2\cdot\sec(2x)\cdot\tan(2x) - \csc(5x)\cdot\cot(5x)\cdot 5$

(expression)$^{\text{number}}$

(ugly)^{number} $\quad u^p \xrightarrow{\text{deriv.}} p\cdot u^{p-1}\cdot u'$

11. Find the derivatives of the following functions.

a) $f(x)=\left(x^2+1\right)^4$

$f'(x) = 4(x^2+1)^3 \cdot 2x$ (deriv. of "ugly")

b) $f(x)=\left(x^3-4x+1\right)^5$

$f'(x) = 5\cdot(x^3-4x+1)^4\cdot(3x^2-4)$

c) $f(x)=5\sqrt{x^3+x^2+4}$

$f'(x) = 5\cdot\dfrac{(3x^2+2x)}{2\cdot\sqrt{x^3+x^2+4}}$ (deriv. of inside)

$(\sqrt{u})' = \dfrac{u'}{2\sqrt{u}}$

d) $f(x)=\sqrt[3]{5x+1}$

$f(x) = (5x+1)^{1/3}$

$f'(x) = \frac{1}{3}(5x+1)^{-2/3}\cdot(5)$

say $f(x)=x^2+1$ $\quad f'(x)=?$

Ans

$f(x) = x^2 + 1$

$\square = 2x$.

$f(x) = \_\_\_\_$.

$f'(x) = \_\_\_\_$

12. Find the derivatives of the following functions.

a) $f(x) = 2x^3 \sin(10x)$

Product rule

chain rule

$f'(x) = 6x^2 \cdot \sin(10x) + 2x^3 \cdot \cos(10x) \cdot 10$

b) $f(x) = 5x\cos\left(4x^2\right)$

Product rule

$f'(x) = 5 \cdot \cos(4x^2) + 5x \cdot \left(-\sin(4x^2)\right) \cdot 8x$

c) $f(x) = \dfrac{9x}{x-10}$

Quotient Rule

$f'(x) = \dfrac{9(x-10) - 9x \cdot 1}{(x-10)^2} = \dfrac{9x - 90 - 9x}{(x-10)^2} = \dfrac{-90}{(x-10)^2}$

d) $f(x) = \dfrac{x^2}{5x-2}$

Quotient Rule

$f'(x) = \dfrac{2x \cdot (5x-2) - x^2(5)}{(5x-2)^2}$

$f'(x) = \dfrac{5x^2 - 4x}{(5x-2)^2}$

(expression)$^{\text{number}}$ $u^8 \rightarrow$

13. Find the derivatives of the following functions.

a) $f(x)=10\cos^8(5x^2)$ $= 10[\cos(5x^2)]^8$

$f'(x) = 10 \cdot 8 \cdot [\cos(5x^2)]^7 \cdot (-\sin(5x^2)) \cdot 10x$

b) $f(x)=\tan^4(3x)$ $= [\tan(3x)]^4$

$f'(x) = 4 \cdot [\tan(3x)]^3 \cdot \sec^2(3x) \cdot 3$

c) $f(x)=4\sin^3(6x)$

$4 \cdot [\sin(6x)]^3$

$f'(x) = 4 \cdot 3 \cdot [\sin(6x)]^2 \cdot \cos(6x) \cdot 6$

$f'(x) = 72 \cdot \sin^2(6x) \cdot \cos(6x)$

OR

2nd deriv.

14. (a) Given: $f(x)=\cos(x)+x^3+x^2+4$, find $f''(0)=?$

$f'(x) = -\sin(x) + 3x^2 + 2x$

$f''(x) = -\cos(x) + 6x + 2$

$f''(0) = -1 + 0 + 2 = \boxed{1}$

(b) Given: $y=x^2+\frac{4}{x}+5x$, find $\frac{d^2y}{dx^2}=?$

Leibniz ↓ ↳ ↑ second deriv.

$\frac{dy}{dx} = 2x + 4 \cdot -\frac{1}{x^2} + 5 = 2x - \frac{4}{x^2} + 5$

$\frac{d^2y}{dx^2} = 2 - 4 \cdot (-2) \cdot x^{-3} + 0$

$= 2 + \frac{8}{x^3}$ //

15.(a) Given: $f(x)=5x^3+5x^2+12$, find the points on the curve where the tangent line is horizontal.

$f'(x)=0$ $\quad 15x^2+10x=0$

$5x(3x+2)=0$

$x=0, \quad x=-\frac{2}{3}$

(b) Given: $f(x)=\frac{1}{3}x^3+x^2+12x+1$, find the points on the curve where the tangent line has slope 12.

$f'(x)=12$ & solve

$x^2+2x+12=12$

$x^2+2x=0 \Rightarrow x(x+2)=0$

$x=0, \quad x=-2$

ex:

$f(1)=5$

$f'(1)=4$

$(1,5)$

$y-5=4(x-1)$

16. Given $x^2 + y^3 + 5xy + 4y = 124$ ← implicit diff.

a) Find $\frac{dy}{dx}$.

b) Find the equation of the tangent line to the curve at (2,4).

Deriv. of both sides implicitly:

$$2x + 3y^2 \cdot y' + (5y + 5x \cdot y') + 4 \cdot y' = 0$$

$$(3y^2 + 5x + 4) \cdot y' = -2x - 5y$$

$$\Rightarrow \boxed{y' = \frac{-2x - 5y}{3y^2 + 5x + 4}}$$

$$\left.\frac{dy}{dx}\right|_{(2,4)} = \frac{-2 \cdot 2 - 5 \cdot 4}{3 \cdot 16 + 5 \cdot 2 + 4} = \frac{-24}{62} = \frac{-12}{31}$$

$m_{tan} = \frac{-12}{31}$ (2,4)

equation: $\boxed{y - 4 = \frac{-12}{31}(x - 2)}$

Exercise:

Find the equation of the tangent line at (2,-1)

Given: $x^2y - y^3 - 4y = 1.$

$2x \cdot y + x^2 \cdot y' - 3y^2 \cdot y' - 4 \cdot y' = 0$

$\Rightarrow \quad (x^2 - 3y^2 - 4) \cdot y' = -2xy$

$\Rightarrow \quad y' = \dfrac{-2xy}{x^2 - 3y^2 - 4}$

$\left.\dfrac{dy}{dx}\right|_{(2,-1)} = \dfrac{-2 \cdot 2 \cdot (-1)}{2^2 - 3 \cdot 1 - 4} = \dfrac{4}{-3} = -\dfrac{4}{3}$

$m = -\dfrac{4}{3}$ Point: $(2,-1)$

equation of tangent line: $y + 1 = -\dfrac{4}{3}(x-2)$

Exercise: Suppose we are given the data in the table about the functions f and g and their derivatives. Find the following values.

x	1	2	3	4
f(x)	3	4	1	2
f'(x)	4	5	2	3
g(x)	2	1	4	3
g'(x)	5	2	3	4

$a)$ $h'(2)$ *if* $h(x) = g(f(x))$

$b)$ $h'(2)$ *if* $h(x) = f(g(x))$

$c)$ $h'(2)$ *if* $h(x) = \dfrac{g(x)}{f(x)}$

$d)$ $h'(2)$ *if* $h(x) = [f(x)]^2 g(x)$

Exercise: Find the points of discontinuity and classify them.