

# **Math 2413- Calculus I**

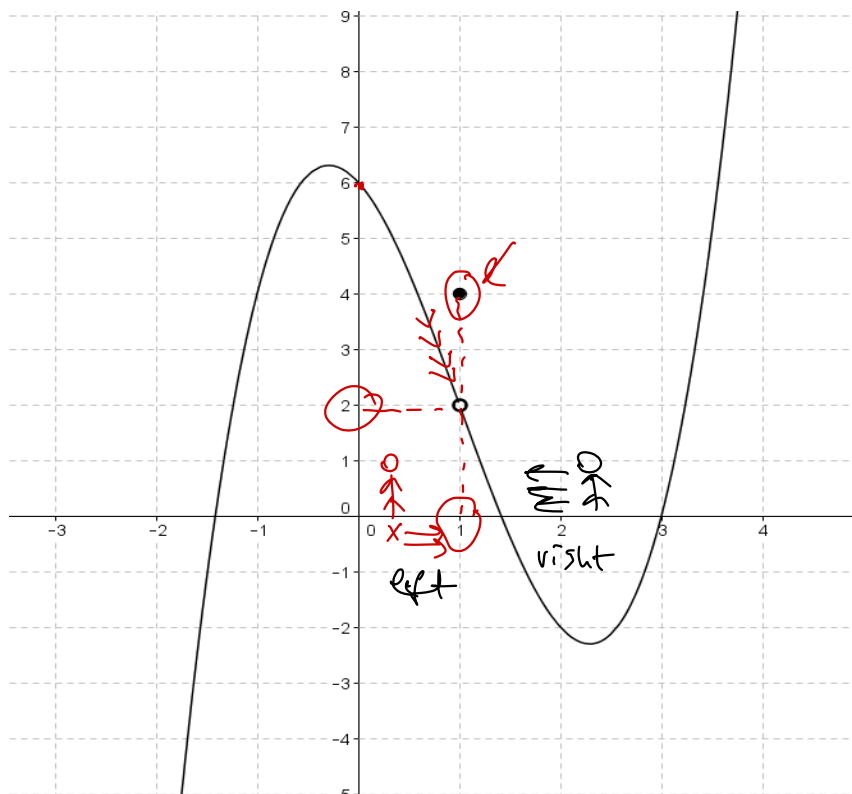
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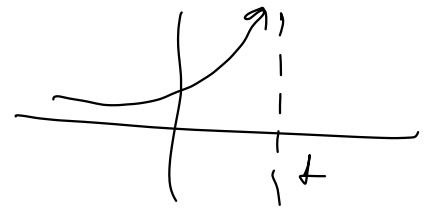
- Check CASA calendar for due dates.
- Bring “blank notes” to class. Completed notes will be posted after class.
- Do your best to attend every lecture and lab. This is a 4 credit course because of the lab component.
- Study after every lecture; work on the quiz covering the topic we cover on the lecture immediately afterwards. Retake your quizzes for more practice.
- Get help when you need help; bring your questions to the labs, or my office hours. We also have tutoring options on campus.
- Make sure you are a member of our team; check the discussion channel for announcements. You can post questions there. Make sure MS teams notifications are ON so that you are notified when we make announcements there.
- When you email me, include course info in the subject line.
- Respect your friends in class; stay away from distractive behavior. Do your best to concentrate on the lecture.

## Section 1.2 – An Intuitive Introduction to Limits

Suppose that a function  $f$  has the following graph.



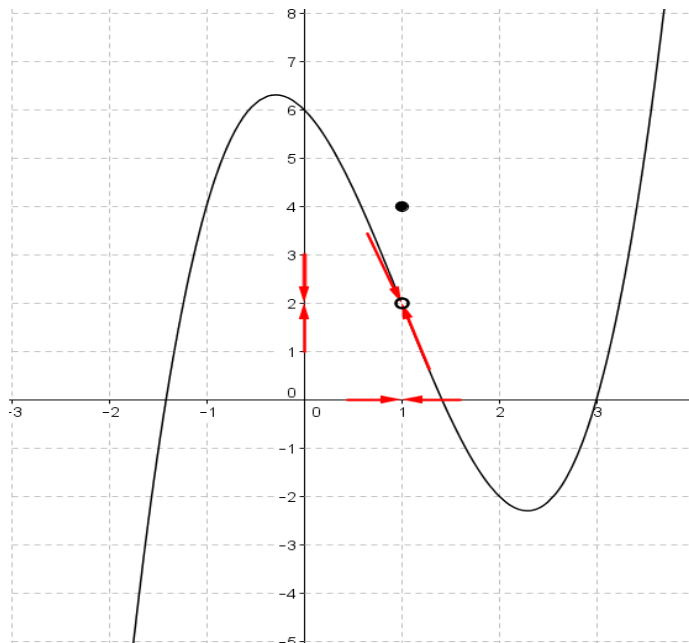
$$f(1) = 4$$



We want to describe the behavior of  $f$  when  $x$  is very close to 1.

- As  $x$  approaches 1 **from the left** (that is,  $x$  is very close to 1 but  $x < 1$ ), what function value do we expect to get?
- As  $x$  approaches 1 **from the right** (that is,  $x$  is very close to 1 but  $x > 1$ ), what function value do we expect to get?
- As  $x$  approaches 1, what function value do we expect to get?

The question is; as  $x$  approaches 1 (symbolized as:  $x \rightarrow 1$ ), is there a **target number** that  $f(x)$  is approaching?



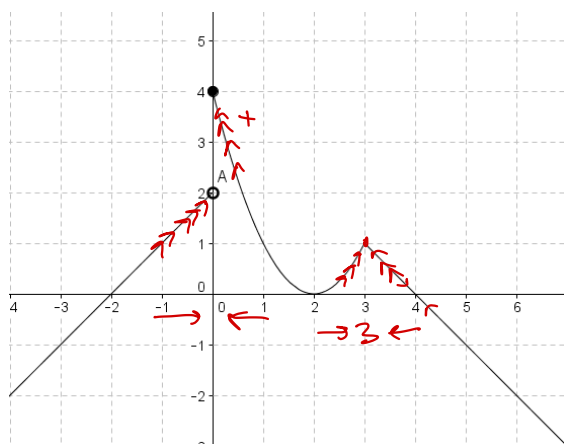
$$\lim_{x \rightarrow c} f(x)$$

We say that 2 is *the limit of  $f(x)$  as  $x$  approaches 1*. This is written as:

**Informal Definition:** We say that the limit of  $f(x)$  as  $x$  approaches  $c$  is the real number  $L$ , if the  $y$ -coordinates of the points  $(x, f(x))$  are getting closer and closer to a certain target number  $L$  as  $x$  approaches  $c$  from each side of  $c$ . This is written as:

$$\lim_{x \rightarrow c} f(x) = L$$

Example:



$$\lim_{x \rightarrow 3} f(x) = \underline{\underline{1}}$$

$$\lim_{x \rightarrow 4} f(x) = 0$$

$$\lim_{x \rightarrow 0} f(x) =$$

Does not exist  
DNE

We can describe the behavior of  $f(x)$  as  $x$  approaches 0 in terms of **one-sided limits**.

Here, 2 is the limit of  $f(x)$  as  $x$  approaches 0 from the left (or from below):

Notation:  $\lim_{x \rightarrow 0^-} f(x) = 2$

$x < 0$

And, 4 is the limit of  $f(x)$  as  $x$  approaches 0 from the right (or from above):

Notation:  $\lim_{x \rightarrow 0^+} f(x) = 4$

$x > 0$

$x \rightarrow c^+$  means  $x > c$

$x \rightarrow c^-$  means  $x < c$

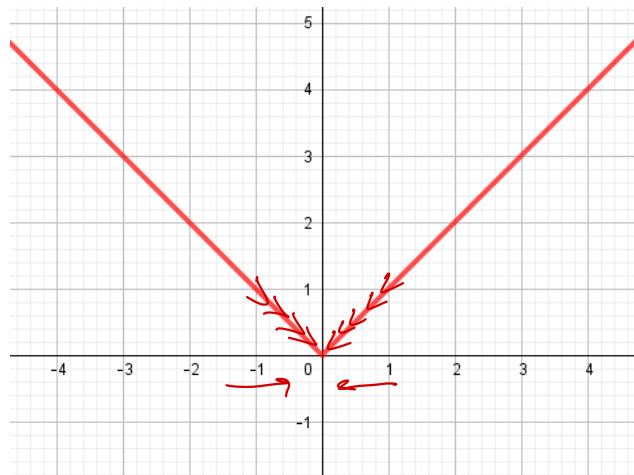
This example illustrates a very important fact about the existence of limit.

**Fact:**  $\lim_{x \rightarrow c} f(x)$  exists if and only if  $\lim_{x \rightarrow c^-} f(x)$  and  $\lim_{x \rightarrow c^+} f(x)$  both exist and are equal.

Example: Here is the graph of  $f(x) = |x|$ . Note that this function is equivalent to:

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

$\rightarrow \lim_{x \rightarrow 0^+} f(x) = 0$   
 $\lim_{x \rightarrow 0^-} f(x) = 0$   
 Hence,  $\lim_{x \rightarrow 0} f(x) = 0$



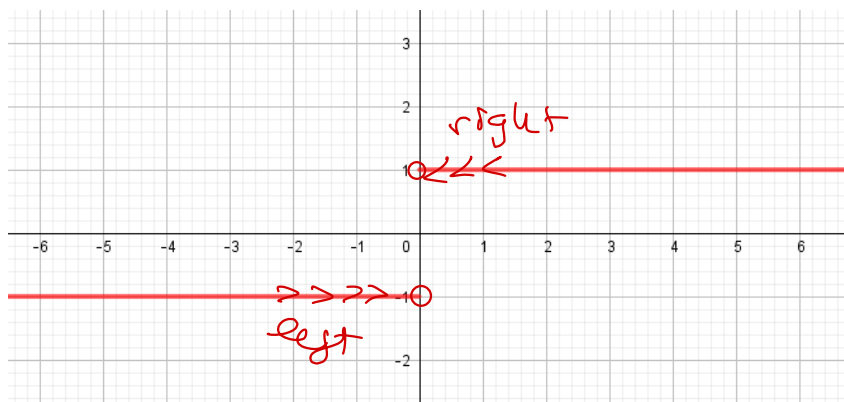
Example: Another function you need to familiar with:

$f(x) = \frac{|x|}{x}$ . This function can be written as:  $\frac{|x|}{x} = \begin{cases} 1, & \text{if } x > 0 \\ -1, & \text{if } x < 0 \end{cases}$

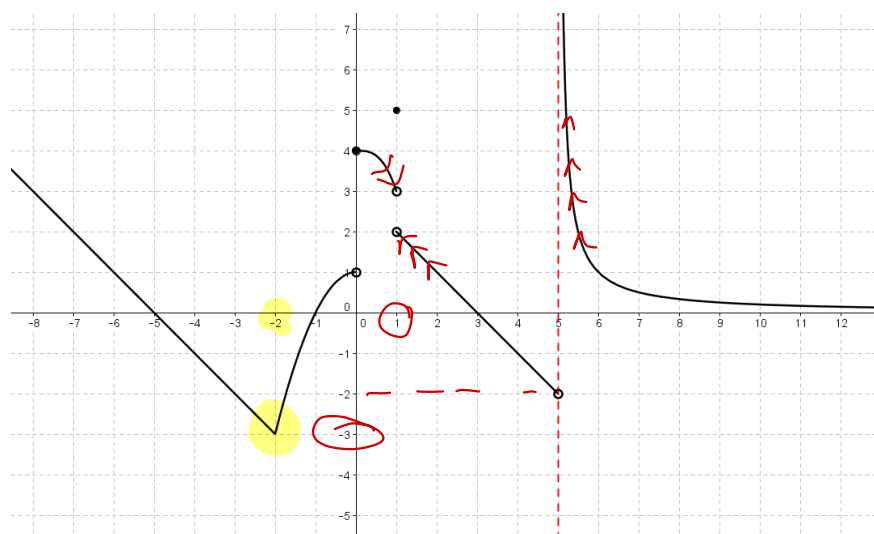
$$\lim_{x \rightarrow 0^+} f(x) = 1$$

$$\lim_{x \rightarrow 0^-} f(x) = -1$$



Hence,  $\lim_{x \rightarrow 0} f(x)$ : DNE

**Example:** Given the graph of  $f$ , evaluate the following limits, if they do exist.



$$\lim_{x \rightarrow -2} f(x) = -3$$

$$\lim_{x \rightarrow 0^+} f(x) = 4$$

$$\lim_{x \rightarrow 0^-} f(x) = 1$$

$$\lim_{x \rightarrow 0} f(x) = \text{DNE}$$

$$\lim_{x \rightarrow 5^-} f(x) = -2$$

$$\lim_{x \rightarrow 5^+} f(x) = \text{DNE}$$

$$\lim_{x \rightarrow 5} f(x) = \text{DNE}$$

$$\lim_{x \rightarrow 1^+} f(x) = 2$$

$$\lim_{x \rightarrow 1^-} f(x) = 3$$

$$\lim_{x \rightarrow 1} f(x) = \text{DNE}$$

$$f(1) = 5$$

Homework: Read Sections 1.1 and 1.2 from your textbook.

Homework #1 is posted on CASA; due in LAB.

Check CASA calendar regularly for announcements and due dates.

Make sure you are a member of our team; check the discussion channel for announcements. You can post questions there.