Math 2413- Calculus I

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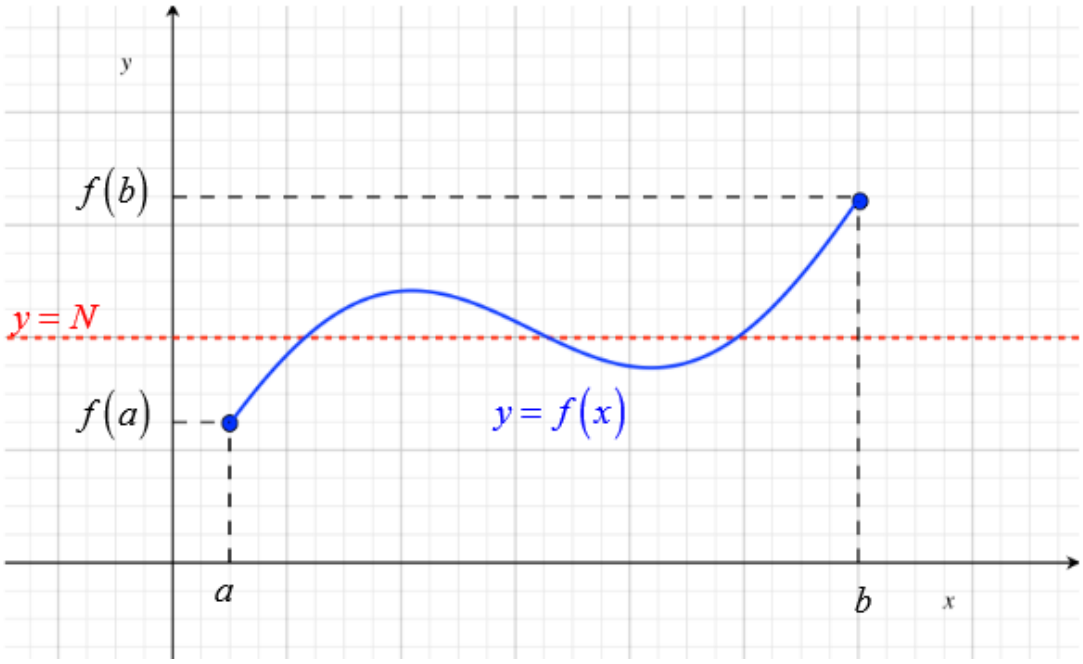
- Check CASA calendar for due dates.
- Bring "blank notes" to class. Completed notes will be posted after class.
- Do your best to attend every lecture and lab. This is a 4 credit course because of the lab component.
- Study after every lecture; work on the quiz covering the topic we cover on the lecture immediately afterwards. Retake your quizzes for more practice.
- Get help when you need help; bring your questions to the labs, or my office hours. We also have tutoring options on campus.
- Make sure you are a member of our team; check the discussion channel for announcements. You can post questions there. Make sure MS teams notifications are ON so that you are notifies when we make announcements there.
- Respect your friends in class; stay away from distractive behavior. Do your best to concentrate on the lecture. Be considerate of others.
- If you email me, mention the course code in the subject line.

Section 1.5 – Intermediate Value Theorem

An important property of continuous functions is given in the following theorem.

Theorem: The Intermediate Value Theorem

If f is a continuous function on the closed interval $[a,b]$, and N is a real number such that $f(a) \le N \le f(b)$, then there is at least one number c in the interval (a,b) such that $f(c) = N$.



That is, if $y = N$ is a horizontal line between $y = f(a)$ and $y = f(b)$, and if f is continuous on $[a,b]$ then, the graph of f must intersect the line $y = N$ at least once.

Example: Given: $f(x) = x^3 - x + 2$.

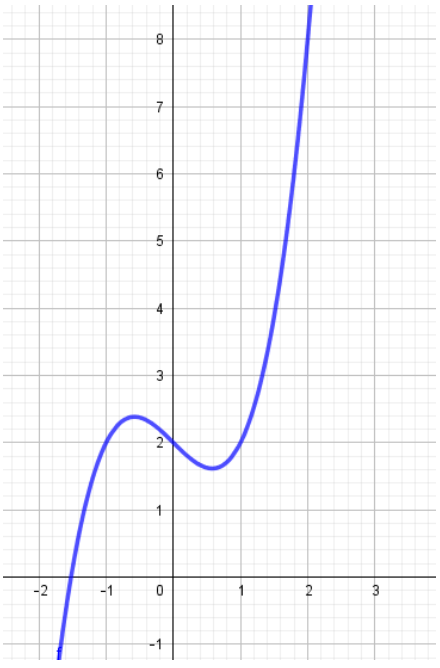
Show that there is a value between 0 and 2 so that $f(x) = 3$.

Conditions of IVT:

(1) Is f continuous on $[a,b]$?

(2) Is N between $f(a)$ and $f(b)$?

Then, there is at least one value c between a and b such that $f(c) = N$.



Exercise: Show that the following equation has a solution in the interval $\left[0,\frac{\pi}{4}\right]$:

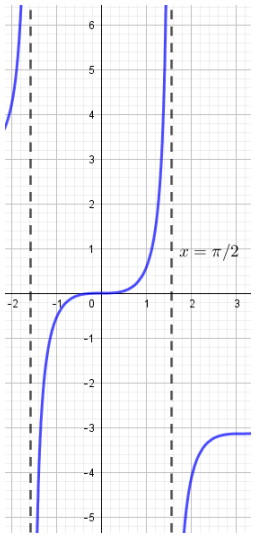
$$2tan(x)-x=1.$$

Conditions of IVT:

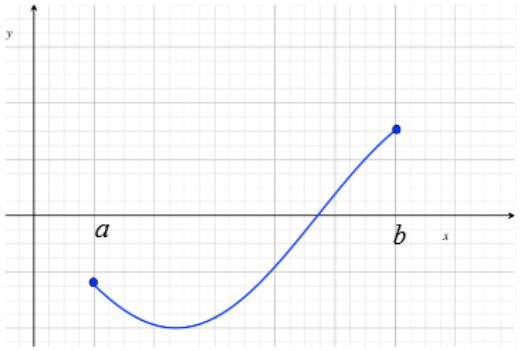
(1) Is f continuous on $[a,b]$?

(2) Is N between $f(a)$ and $f(b)$?

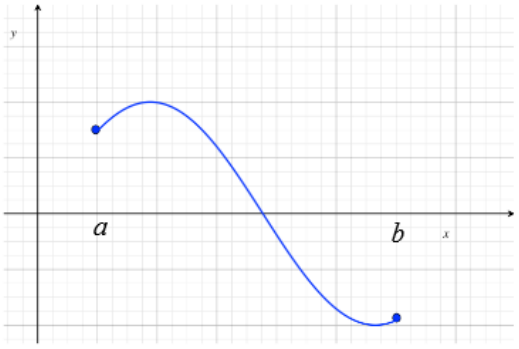
Then, there is at least one value c between a and b such that $f(c)=N$.



Remark: We can use the Intermediate Value theorem to prove the existence of roots (zeros or $x-$intercepts) of a function.



$f(a)<0<f(b)$



$f(b)<0<f(a)$

Example: Use the Intermediate Value theorem to show that the function has a root in the indicated interval.

$f(x)=x^2-4x+3, \quad [0,1].$

Conditions of IVT:

(1) Is f continuous on $[a,b]$?

(2) Is N between $f(a)$ and $f(b)$?

Then, there is at least one value c between a and b such that $f(c)=N$.

Find the value of c:

Example: Does IVT guarantee a solution for $f(x)=0$ over the interval $\left(0,\frac{\pi}{2}\right)$?

$f(x)=2\sin(x)-\cos(x)-4x^2$

Check the conditions of IVT:

1) Is f continuous on $[a,b]$?
2) Is N between $f(a)$ and $f(b)$?

Example: Does IVT guarantee a solution for $f(x)=0$ over the interval $(1,5)$?

$f(x)=\frac{x+1}{x-4}$

Check the conditions of IVT:

1) Is f continuous on $[a,b]$?
2) Is N between $f(a)$ and $f(b)$?

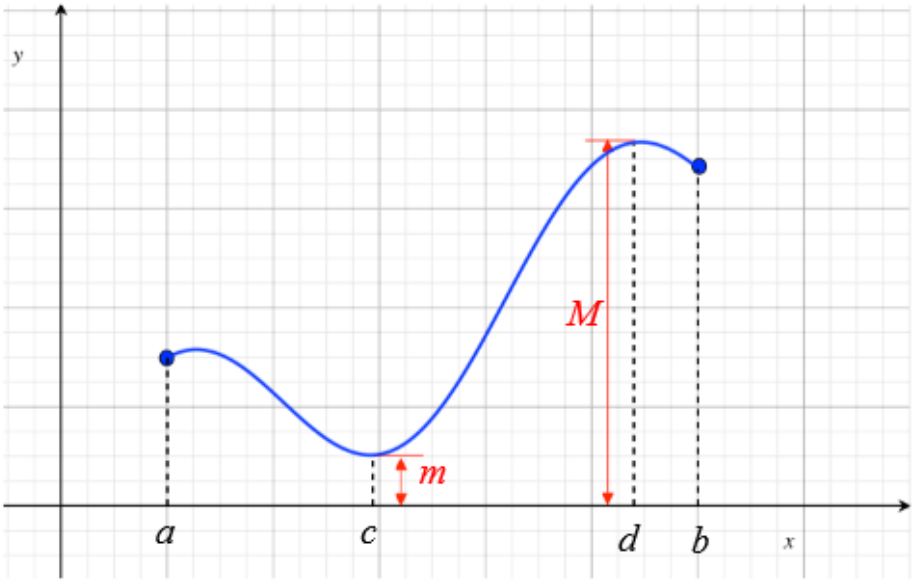
Exercise: Can you graph a function satisfying the properties below? If yes, graph one. If no, state why.

- Continuous on [0,5]
- $f(0)=1$, $f(2)=3$, $f(5)=-1$
- The function does not intersect the x-axis.

Another property of continuous functions is about extreme values.

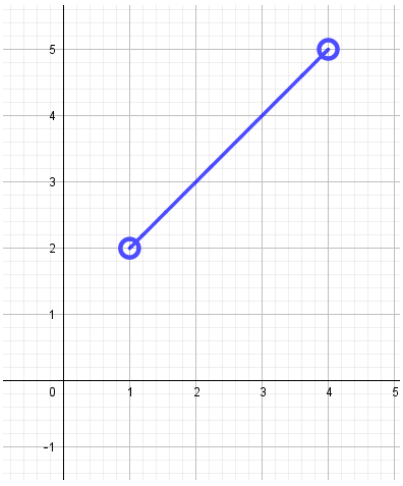
Theorem 1.5.2: The Extreme-Value Theorem

If f is continuous on a bounded interval $[a,b]$, then f takes on both a maximum value and a minimum value.



Example: Does the following function have a maximum and a minimum value over the interval [1,4]?

Which condition of EVT is not satisfied?



Example: Does the following function have a maximum and a minimum value over the interval [1,4]?

Which condition of EVT is not satisfied?