Completed Notes

Math 2413- Calculus I

Dr. Melahat Almus

Email: malmus@uh.edu

- Check CASA calendar for due dates.
- Bring “blank notes” to class. Completed notes will be posted after class.
- Do your best to attend every lecture and lab. This is a 4 credit course because of the lab component.
- Study after every lecture; work on the quiz covering the topic we cover on the lecture immediately afterwards. Retake your quizzes for more practice.
- Get help when you need help; bring your questions to the labs, or my office hours. We also have tutoring options on campus.
- Make sure you are a member of our team; check the discussion channel for announcements. You can post questions there. Make sure MS teams notifications are ON so that you are notifies when we make announcements there.
- Respect your friends in class; stay away from distractive behavior. Do your best to concentrate on the lecture. Be considerate of others.
- If you email me, mention the course code in the subject line.

→ Day 4: S1.5 & S1.6

Quiz 2: S1.3

Quiz 3: S1.3

Quiz 4: S1.4 & S1.5

Quiz 5: S1.6

Remember:
1 credit hour
calls for 3 hrs/week
of studying;

2413 is a 4 credit
course; $4 \cdot 3 = 12$ hours/week

expected
study time
per week

12 hrs

IVT

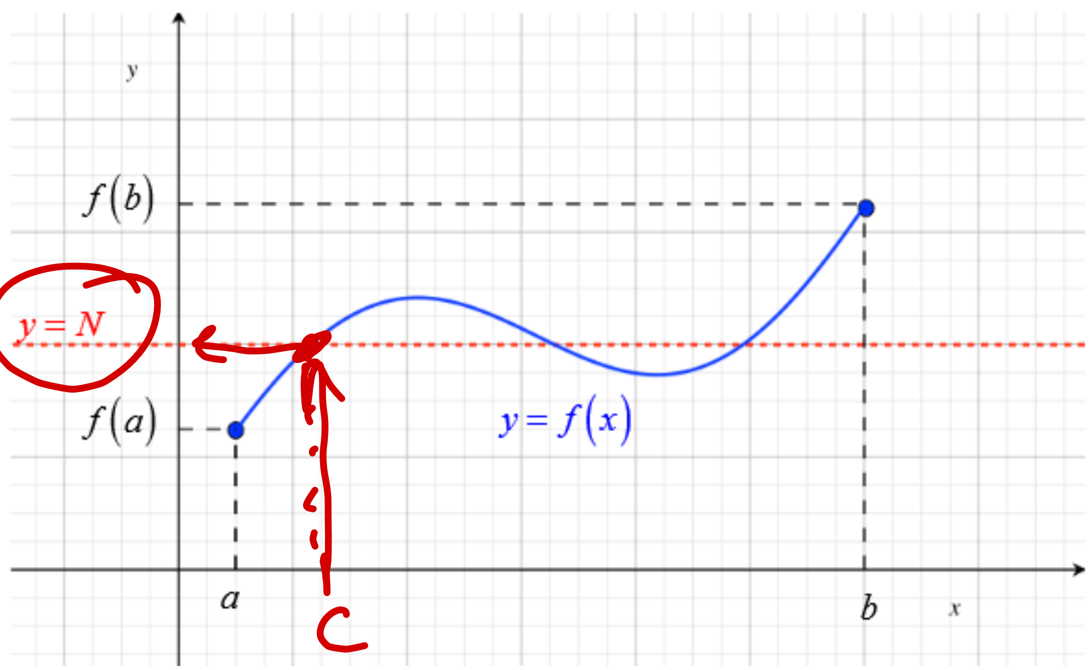
Section 1.5 – Intermediate Value Theorem

An important property of continuous functions is given in the following theorem.

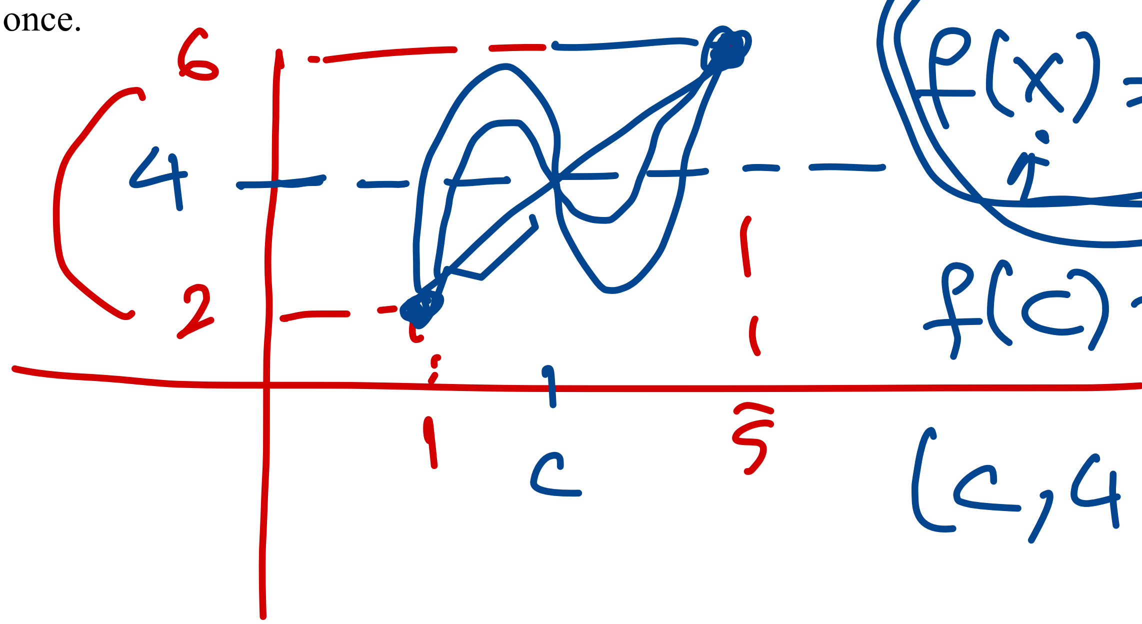
Theorem: The Intermediate Value Theorem

If f is a continuous function on the closed interval $[a,b]$, and N is a real number such that $f(a) \le N \le f(b)$, then there is at least one number c in the interval (a,b) such that $f(c) = N$.

$f: [a,b] \to [\quad]$?



That is, if $y = N$ is a horizontal line between $y = f(a)$ and $y = f(b)$, and if f is continuous on $[a,b]$ then, the graph of f must intersect the line $y = N$ at least once.



$f(x) = 4$

$f(c) = 4$

$(c, 4)$

$x^3 - x + 2 = 3$

Example: Given: $f(x) = x^3 - x + 2$.

Show that there is a value between 0 and 2 so that $f(x) = 3$.

$[0, 2]$

Conditions of IVT:

(1) Is f continuous on $[a,b]$? ✓

(2) Is N between $f(a)$ and $f(b)$?

$f(0)$ $f(2)$

$f(0) = 2$

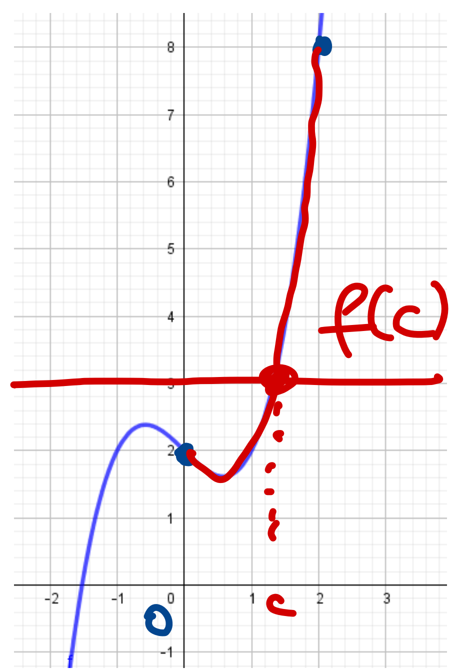
$f(2) = 8 - 2 + 2 = 8$

$2 < 3 < 8$

Then, there is at least one value c between a and b such that $f(c) = N$.

between 0 & 2 such that

$f(x) = 3$



$f(c) = 3$

Exercise: Show that the following equation has a solution in the interval $\left[0, \frac{\pi}{4}\right]$:

$$2\tan(x) - x = 1.$$

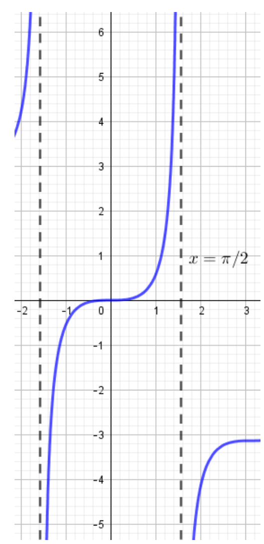
Conditions of IVT:

(1) Is f continuous on $[a,b]$?

(2) Is N between $f(a)$ and $f(b)$?

Then, there is at least one value c between a and b such that $f(c) = N$.

$-\pi/2$ $\quad \pi/2$

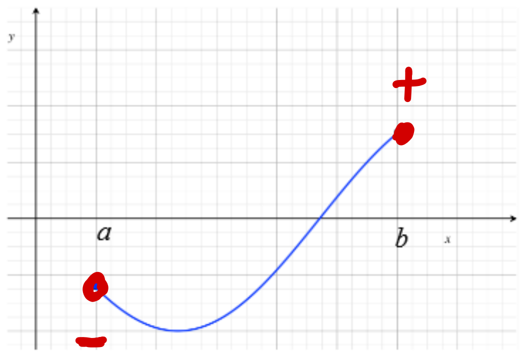


ex: $f(x) = \dfrac{1}{x-1}$ on $[0, 2]$ ✗ cts ✗

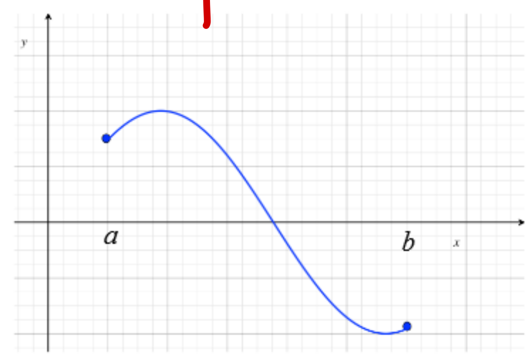
$f(x) = \dfrac{1}{x-1}$ on $[2, 4]$ cts ✓

x-intercept is a solution to $f(x)=0$

Remark: We can use the Intermediate Value theorem to prove the existence of roots (zeros or $x-$intercepts) of a function.



$f(a)<0<f(b)$

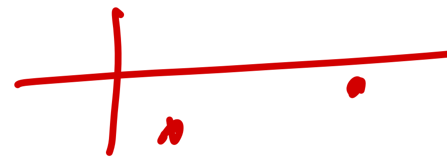


$f(b)<0<f(a)$

Example: Use the Intermediate Value theorem to show that the function has a root in the indicated interval.

$f(x)=x^2-\cancel{4}x+3$, $[0,1]$.

$6x$



Conditions of IVT: $[0,1]$

(1) Is f continuous on $[a,b]$? ✓

(2) Is N between $f(a)$ and $f(b)$?

$f(0) = 0 - 0 + 3 = 3 \quad (+)$

$f(1) = 1 - 6 + 3 = -2 \quad (-)$

Then, there is at least one value c between a and b such that $f(c)=N$.

0 & 1 $\quad f(c)=0$.

Find the value of c:

$f(x)=0$

$x^2-6x+3=0$

$x_{1,2} = \dfrac{6 \pm \sqrt{36-4 \cdot 3}}{2 \cdot 1}$

pick the one that is in $[0,1]$

Example: Does IVT guarantee a solution for $f(x)=0$ over the interval $\left(0,\frac{\pi}{2}\right)$?

$f(x)=2\sin(x)-\cos(x)-4x^2$

No

Check the conditions of IVT:

1) Is f continuous on $[a,b]$? $\left[0,\frac{\pi}{2}\right]$? cts ✓
2) Is N between $f(a)$ and $f(b)$?

$f(0)=2\cdot 0-1-0=-1$

$f(\pi/2)=2\cdot 1-0-4\cdot\frac{\pi^2}{4}=2-\pi^2$

Example: Does IVT guarantee a solution for $f(x)=0$ over the interval $(1,5)$?

$$\boxed{f(x)=\frac{x+1}{x-4}}$$

rational

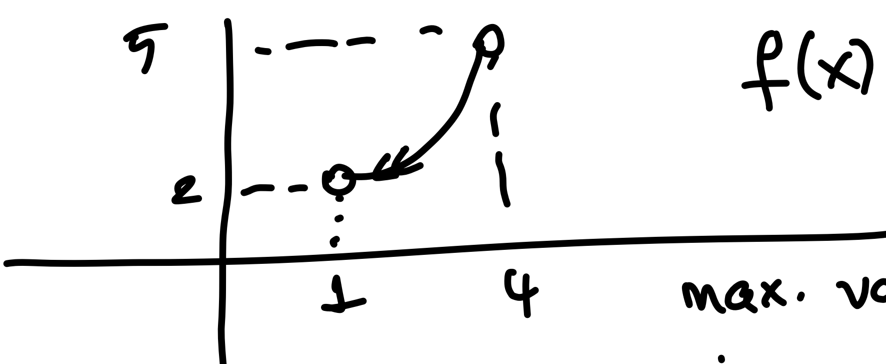
Check the conditions of IVT:

1) Is f continuous on $[a,b]$? $[1,5]$? Not continuous.
2) Is N between $f(a)$ and $f(b)$?

IVT does not apply.

Exercise: Can you graph a function satisfying the properties below? If yes, graph one. If no, state why.

- Continuous on [0,5]
- $f(0)=1$, $f(2)=3$, $f(5)=-1$
- The function does not intersect the x-axis.



$f(x): (1,4)\to(2,5)$

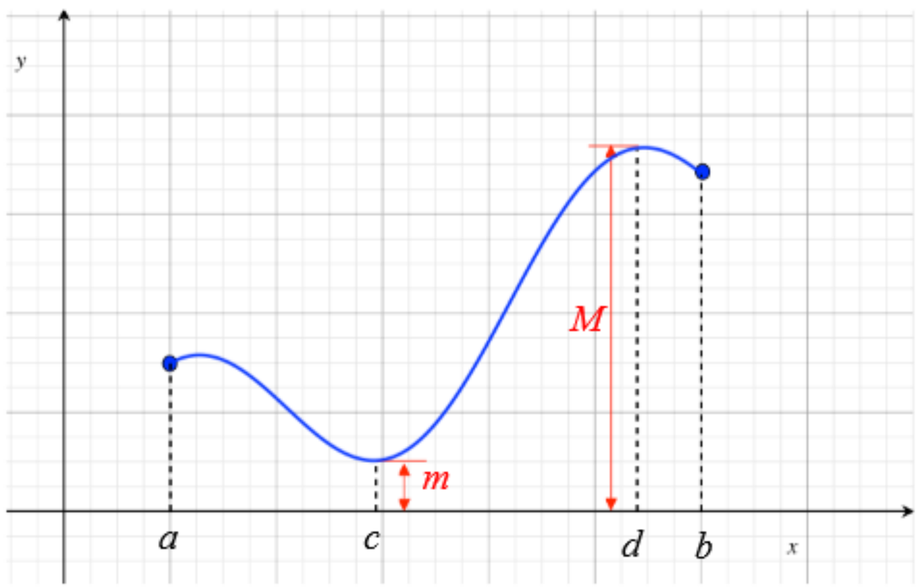
max. value of f(x) ? NO

min value of f(x) ? NO

Another property of continuous functions is about extreme values.

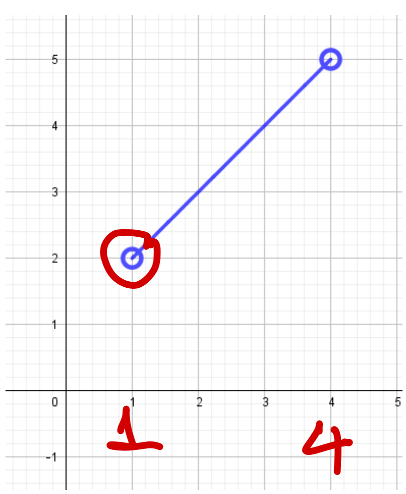
Theorem 1.5.2: The Extreme-Value Theorem

If f is continuous on a bounded interval $[a,b]$, then f takes on both a maximum value and a minimum value.



Example: Does the following function have a maximum and a minimum value over the interval [1,4]?

Which condition of EVT is not satisfied?



max value: None

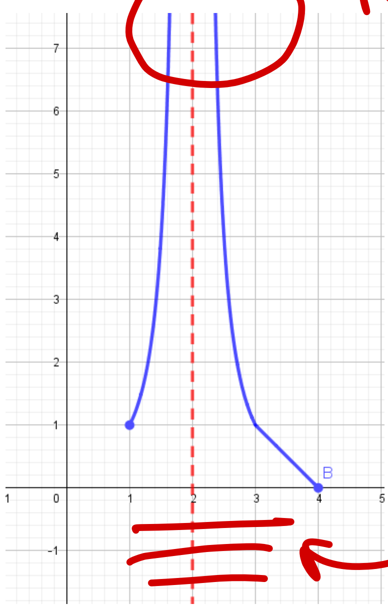
min value: None

$f: \underline{(}1,4\underline{)} \to$

Example: Does the following function have a maximum and a minimum value over the interval [1,4]?

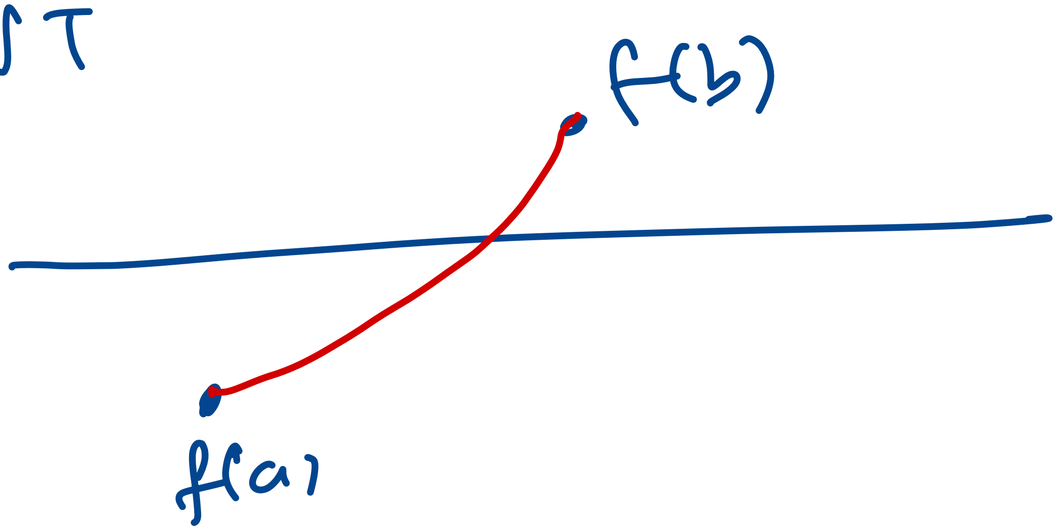
Which condition of EVT is not satisfied?

No max value



Not cts

IVT



$y=0$

x-axis

solution

root

x-intercept

$f(x)=0$