Math 2413- Calculus I

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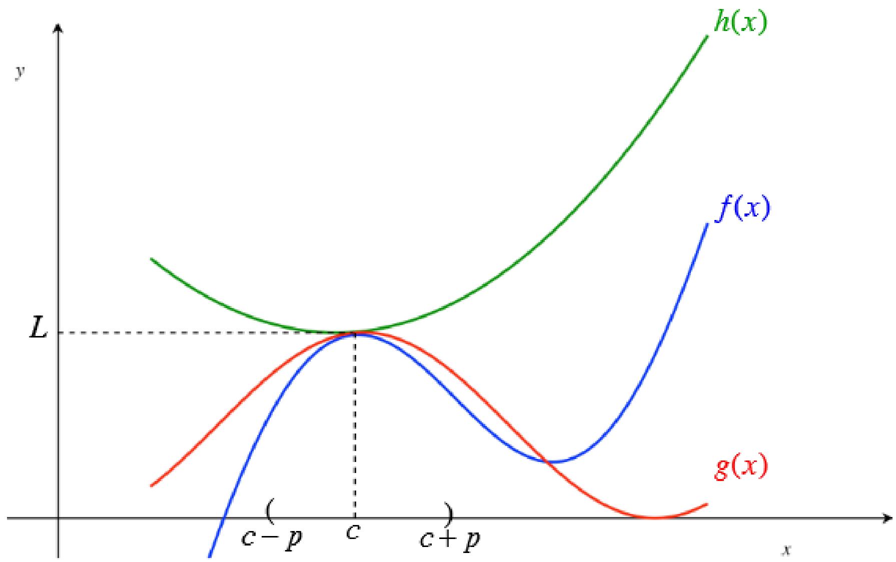
- Check CASA calendar for due dates.
- Bring “blank notes” to class. Completed notes will be posted after class.
- Do your best to attend every lecture and lab. This is a 4 credit course because of the lab component.
- Study after every lecture; work on the quiz covering the topic we cover on the lecture immediately afterwards. Retake your quizzes for more practice.
- Get help when you need help; bring your questions to the labs, or my office hours. We also have tutoring options on campus.
- Make sure you are a member of our team; check the discussion channel for announcements. You can post questions there. Make sure MS teams notifications are ON so that you are notifies when we make announcements there.
- Respect your friends in class; stay away from distractive behavior. Do your best to concentrate on the lecture. Be considerate of others.
- If you email me, mention the course code in the subject line.

Section 1.6 – The Pinching Theorem

Theorem: The Pinching Theorem

Let $p>0$ and c be a real number. Suppose that $f(x) \leq g(x) \leq h(x)$ for any x in the interval $(c-p, c+p)$ (except possibly at $x=c$).

If $\lim\limits_{x \to c} f(x) = \lim\limits_{x \to c} h(x) = L$, then $\lim\limits_{x \to c} g(x) = L$.



Trigonometric Limits:

$$\lim_{x \to 0} \sin(x) =$$

$$\lim_{x \to 0} \cos(x) =$$

In general, $\lim_{x \to c} \sin(x) = \sin(c)$ and $\lim_{x \to c} \cos(c) = \cos(c)$.

That is, step 1 is direct substitution as before.

Example:

$$\lim_{x \to \pi} \left(\frac{1 + 2\cos(x)}{4x} \right) = ?$$

Example:

$$\lim_{x \to \frac{\pi}{2}} \left(\frac{4 + \sin(x)}{2 + \cos(x)} \right) = ?$$

Now, we look at two very important limits.

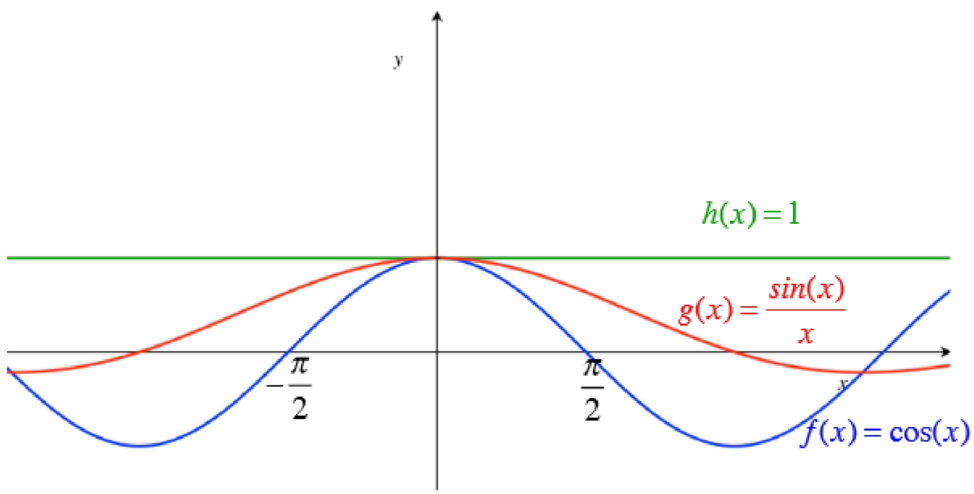
$$\lim_{x \to 0} \frac{\sin(x)}{x} = ? \quad \text{and} \quad \lim_{x \to 0} \frac{1-\cos(x)}{x} = ?.$$

If you try to use direct substitution on these limits, you'll get the indeterminate form 0/0.

Idea: For the function $g(x) = \frac{sin(x)}{x}$, if $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, then $cos(x) \le \frac{sin(x)}{x} \le 1$.

Since $\lim_{x \to 0} cos(x) = 1$ and $\lim_{x \to 0} 1 = 1$, by the Pinching theorem, we conclude that

$$\lim_{x \to 0} \frac{sin(x)}{x} = 1.$$



Fact: $\lim\limits_{x\to 0}\dfrac{sin(x)}{x}=1$ and $\lim\limits_{x\to 0}\dfrac{1-cos(x)}{x}=0$.

Fact: $\lim\limits_{x\to 0}\dfrac{sin(ax)}{bx}=\dfrac{a}{b}$ and $\lim\limits_{x\to 0}\dfrac{1-cos(ax)}{bx}=0$.

ALWAYS pay attention to what value *x* is approaching. These facts only apply in certain cases (0/0).

Examples:

a) $\lim\limits_{x\to 0}\dfrac{sin(5x)}{x}=$

b) $\lim\limits_{x\to 0}\dfrac{sin(2x)}{6x}=$

c) $\lim\limits_{x\to 0}\dfrac{2x}{sin(7x)}=$

Examples:

$$\boxed{\lim_{x\to 0}\frac{1-\cos(x)}{x}=0. \qquad \lim_{x\to 0}\frac{1-\cos(ax)}{bx}=0.}$$

a) $\lim\limits_{x\to 0}\dfrac{1-\cos(2x)}{5x}=$

b) $\lim\limits_{x\to 0}\dfrac{1-\cos(4x)}{9x}=$

Examples:

a) $\lim\limits_{x\to 0}\dfrac{\sin^2(4x)}{5x}=$

b) $\lim\limits_{x\to 0}\dfrac{\sin(4x)}{\sin(9x)}=$

Examples:

a) $\lim\limits_{h \to 0} \dfrac{\sin\left(h^2\right)}{5h^2} =$

b) $\lim\limits_{x \to 0} \dfrac{5x^2}{\sin^2(6x)} =$

c) $\lim\limits_{t \to 0} \dfrac{\sin\left(5t^2\right)}{t^2 \cos(2t)} =$

Examples:

a) $\lim\limits_{x\to 0}\dfrac{\tan(5x)}{4x}=$

b) $\lim\limits_{x\to 0}\dfrac{\tan(6x)}{\tan(4x)}=$

Sometimes, trig identities might be needed to simplify the given expression.

Examples:

a) $\lim\limits_{x\to 0} \frac{1-\cos^2(2x)}{7x} =$

b) $\lim\limits_{x\to 0} x^2 \csc\left(5x^2\right) =$

c) $\lim\limits_{x\to 0} \frac{\cos(x)\tan(x)}{x} =$

d) $\lim\limits_{x\to 0} \frac{1-\sec^2(4x)}{x^2} =$

ALWAYS pay attention to what value the variable is approaching; plug in first!

Review identities and unit circle from Precalculus.

Example:

$$\lim_{x\to\pi/4}\frac{\tan(3x)}{8x}=$$

Exercise: $\displaystyle \lim_{x\to\pi}\frac{\sin(x)}{x-\pi}=$

NEXT: Chapter 2.