Math 2413- Calculus I

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- Check CASA calendar for due dates.
- Bring "blank notes" to class. Completed notes will be posted after class.
- Do your best to attend every lecture and lab. This is a 4 credit course because of the lab component.
- Study after every lecture; work on the quiz covering the topic we cover on the lecture immediately afterwards. Retake your quizzes for more practice.
- Get help when you need help; bring your questions to the labs, or my office hours. We also have tutoring options on campus.
- Respect your friends in class; stay away from distractive behavior. Do your best to concentrate on the lecture.
- If you email me, mention the course code in the subject line. Email is the best way to communicate with me outside of class. Teams chat messages are not monitored or replied to.

Warm up questions covering last section. Answer them before class.

Question# If $f(0)=2$ and $f'(0)=4$, what is the equation of the tangent line at $x=0$?

Question# Is $f(x)=\dfrac{1}{x^2}$ differentiable at $x=0$?

Chapter 2

Section 2.2 – Derivatives of Polynomials and Trig Functions

Finding derivatives using the limit definition can be quite tedious. In this section, we cover some rules that we can use to compute derivatives.

Fact: If $f(x)=k$, where k is a real number, then $f'(x)=0$.

Reason: The graph of a constant function is a horizontal line with slope 0.

Fact: If $f(x)=x$, then $f'(x)=1$

Reason: The rate of change of this function is equal to its slope (since the equation is linear).

Or: $$\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)=\lim_{h\to 0}\left(\frac{(x+h)-(x)}{h}\right)=\lim_{h\to 0}\left(\frac{h}{h}\right)=\lim_{h\to 0}(1)=1$$

In general, for any linear function with slope *m*, the derivative is *m*.

$$f(x)=mx+b \rightarrow f'(x)=m$$

Now, here's the formula to find the derivatives of power functions:

RULE #1: Power Rule

If $f(x) = x^n$, where n is a real number, then: $f'(x) = nx^{n-1}$.

Examples:

$f(x) = x^2; \quad f'(x) =$

$f(x) = x^3; \quad f'(x) =$

$f(x) = x^4; \quad f'(x) =$

$f(x) = \frac{1}{x}; \quad f'(x) =$

$f(x) = \frac{1}{x^2}; \quad f'(x) =$

$f(x) = \sqrt{x}; \quad f'(x) =$

Theorem: Let k be a real number. If f and g are differentiable at x, then so are $f+g$, $f-g$ and $k\cdot f$. Moreover,

- $(f \pm g)'(x) = f'(x) \pm g'(x)$,
- $(k \cdot f)'(x) = k \cdot f'(x)$.

Example:

$f(x) = 5x^2$; $f'(x) =$

Example:

$f(x) = x^3 + 4$; $f'(x) =$

Theorem: Derivatives of Polynomials

Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_2 x^2 + a_1 x + a_0$ be a polynomial function. The derivative is:

$$f(x) = a_n \cdot n \cdot x^{n-1} + a_{n-1} \cdot (n-1) \cdot x^{n-1} + ... + a_2 \cdot 2x + a_1.$$

That is, to describe this informally: we apply the power rule to each term, and "keep" the coefficients.

Example:

$f(x) = x^3 + x^2 + x + 2$; $\quad f'(x) =$

Example:

$f(x) = 5x^{10} - 6x^3 + 12$; $\quad f'(x) =$

Example:

$f(x) = x^{10} - \frac{1}{2}x^6 + 5x$; $\quad f'(1) =$

What if the function is not a polynomial? We can still use the same idea; work with each term:

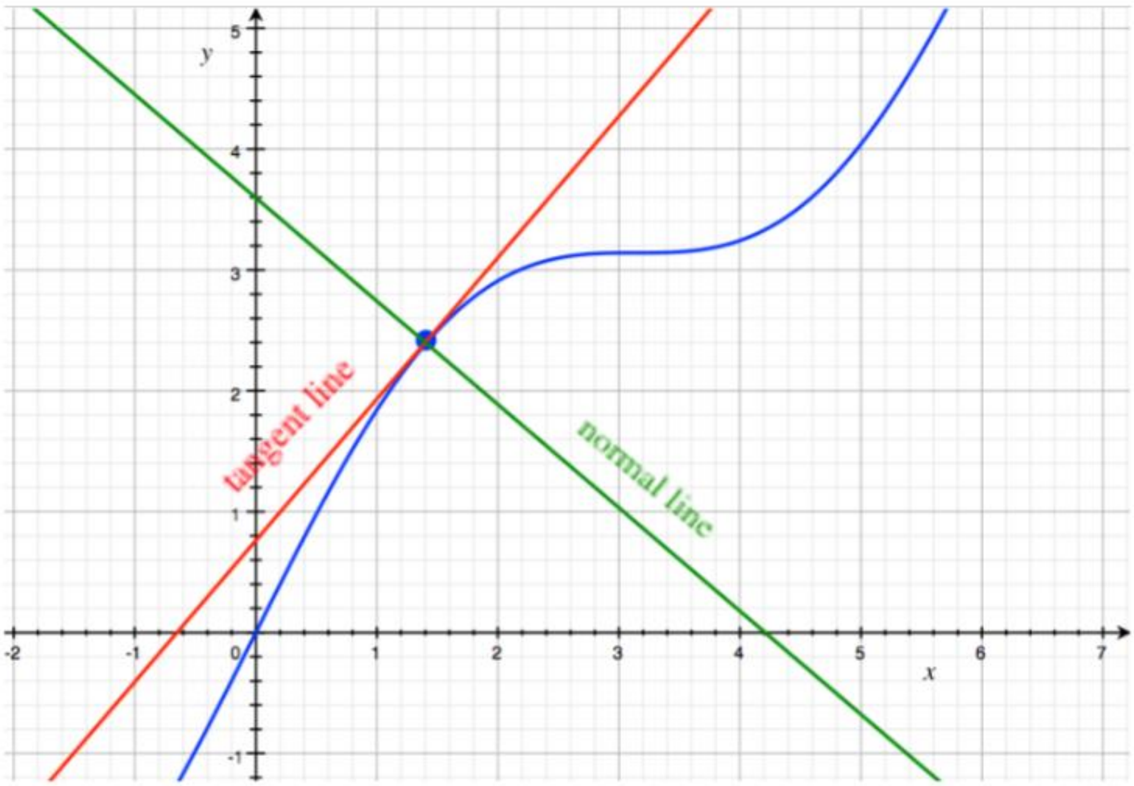
Example:

$f(x) = 4x^3 + 2\sqrt{x} + \frac{1}{x} + \frac{2}{x^2}$; $\quad f'(x) =$

Example: $\quad f(x) = \frac{x^5 + 10x^3 + 4}{x^2}$; $\quad f'(x) =$

Exercise: Given $f'(x) = 4x^3 + 3x^2 + 2x + 5$, what can be the rule for $f(x)$?

Tangent and Normal lines:



Slope of the tangent line at $x = a$ = Derivative of the function at $x = a$.

$$m_{\text{tangent}} = f'(a) \qquad \text{and} \qquad m_{\text{normal}} = -\frac{1}{m_{\text{tangent}}} = -\frac{1}{f'(a)}.$$

Example: Find the equation of the line **<u>tangent</u>** to the curve

$f(x) = 5x^2 + 2x$ at $x = 2$.

Example: Find the equation of the line **<u>normal</u>** to the curve

$f(x) = x^3 + x$ at $x = 1$.

Example: Find the **slope** of the line **normal** to the curve

$f(x) = x + \frac{20}{x}$ at $x = 4$.

In some cases, you will be given a certain slope and be asked to find the points where the tangent line has that slope. Find the derivative, set it equal to the given value and solve.

Example: Consider the function $f(x) = 3x^4 - 6x^2 + 5$. Find the points where the tangent line is horizontal.

Example: Consider the function $f(x) = x^2 + x + 2$. Find the points where the normal line has slope 1/5.

Derivatives of the Six Trig Functions:

$$(\sin(x))' = \cos(x)$$
$$(\cos(x))' = -\sin(x)$$
$$(\tan(x))' = \sec^2(x)$$
$$(\cot(x))' = -\csc^2(x)$$
$$(\sec(x))' = \sec(x)\cdot\tan(x)$$
$$(\csc(x))' = -\csc(x)\cdot\cot(x)$$

These formulas are derived using the definition of derivative and limit facts involving trig functions (and in come cases sum and difference formulas are also used). You can read your book to see how these formulas are derived.

Review unit circle; you will need this to answer derivative questions.

Example: For $f(x)=10\sin(x)$, find $f'\left(\frac{\pi}{6}\right)$.

Example: For $f(x) = 2\cos(x) + 4\sin(x)$, find $f'\left(\frac{\pi}{4}\right)$.

Example: For $f(x) = 6\tan(x) - \sin(x) + x$, find $f'(x)$.

Example: Consider the function $f(x) = 2\sin(x) + 1$ over $[0, 2\pi]$, find the points where the tangent line is horizontal.

Exercise: Consider the function $f(x) = 2\cos(x)$ over $[0, 2\pi]$, find the points where the **tangent** line has slope 1.

Leibniz's d/dx Notation

The derivative of a function y can be denoted as

$\frac{dy}{dx}$, if y is a function is in terms of x,

$\frac{dy}{dt}$, if y is a function is in terms of t,

and so on. Here, $\frac{dy}{dx}$ indicates the derivative of y with respect to x.

If $y = x^2$, then $\frac{dy}{dx} =$.

If $y = t^2$, then $\frac{dy}{dt} =$.

If $w = 5z^3 + z$, then we may take the derivative of w with respect to z: $\frac{dw}{dz} =$

The double-d notation may also be used as a prefix to the function to be differentiated:

$$\frac{d}{dx}(\text{expression}) = \text{the derivative of "expression" with respect to } x.$$

Example: $\frac{d}{dx}\left(x + \frac{1}{x}\right) = ?$

Example: $\frac{d}{dt}(5\cos(t)) = ?$

Example: If $y = x^3 + x$, find $\left.\frac{dy}{dx}\right|_{x=2}$.

Exercise: If $y = \sqrt{w} - 2w$, find $\left.\frac{dy}{dw}\right|_{w=1}$.

Higher Order Derivatives:

$f(x) = x^4 + x^3 + x^2 + 2x + 10$

The first derivative of f : $f'(x) =$

The second derivative of f : $f''(x) = \left[f'(x) \right]' =$

The third derivative of f : $f'''(x) = \left[f''(x) \right]' =$

The fourth derivative of f : $f^{(4)}(x) = \left[f'''(x) \right]' =$

The fifth derivative of f : $f^{(5)}(x) = \left[f^{(4)}(x) \right]' =$

In general, $f^{(n)}$ stands for the nth **order derivative of** f.

The functions f', f'', f''', $f^{(4)}, \ldots, f^{(n)}$ are called the derivatives of f of orders $1,2,3,\ldots,n$, respectively.

Remark:

$f^{(4)}(x)$ stands for the fourth order derivative of $f(x)$, while $f^4(x)$ means $\left[f(x) \right]^4$.

To see a variant of this notation, let $y = x^5$;

$y' = 5x^4$, $y'' = 20x^3$, $y''' = 60x^2$, and so on.

With the **double-*d* notation**, the second order derivative is:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) \quad \text{or} \quad \frac{d^2}{dx^2}\left[f(x)\right] = \frac{d}{dx}\left[\frac{d}{dx}\left[f(x)\right]\right],$$

and the third order derivative is:

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) \quad \text{or} \quad \frac{d^3}{dx^3}\left[f(x)\right] = \frac{d}{dx}\left[\frac{d^2}{dx^2}\left[f(x)\right]\right],$$

and so on.

Example: $f(x) = x^4 - x^2 + 5x; \quad f''(1) = ?$

Example: $f(x) = x^5 + x; \quad \frac{d^3f}{dx^3} = ?$

A QUICK REVISIT - S2.1: On differentiability:

Example: Given $f\left(x\right)=\begin{cases} Ax+B, & \text{if } x>1 \\ x^3+8, & \text{if } x\le 1 \end{cases}$; find the values of A and B so that this function is differentiable.

Hint: Check continuity first.