Completed

Math 2413- Calculus I

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- Check CASA calendar for due dates.
- Bring "blank notes" to class. Completed notes will be posted after class.
- Do your best to attend every lecture and lab.
- Study after every lecture; work on the quiz covering the topic we cover on the lecture immediately afterwards. Retake your quizzes for more practice.
- Get help when you need help; bring your questions to the labs, or my office hours. We also have tutoring options on campus.
- Respect your friends in class; stay away from distractive behavior. Do your best to concentrate on the lecture.
- If you email me, mention the course code in the subject line. Email is the best way to communicate with me outside of class. Teams chat messages are not monitored or replied to.

Warm up questions. Answer them before class.

Question# If $f(x)=(4x-3)^5$, $f'(1)=?$

$f'(x)=5(4x-3)^4\cdot 4$

$f'(1)=20$

Question# If $f(x)=\cos^2(4x)$, $f'(0)=?$

$f'(x)=2\cdot\cos(4x)\cdot -\sin(2x)\cdot 4$

Question# If $f(x)=x\sin(5x)$, $f'(0)=?$

$f'(x)=1\cdot\sin(5x)+x\cdot\cos(5x)\cdot 5$

$f'(0)=0+0=0$

$x^2 + x \cdot y + y^3 = 3$

Section 2.4 – Implicit Differentiation

Question: If $x^2 + y^2 = 1$, what is $\frac{dy}{dx}$?

Consider the circle with equation $x^2 + y^2 = 1$; we might be interested in finding the equation of the tangent line at a point on this circle. How would we find the derivative?

$(y^2)' = 2 \cdot y \cdot y'$

How do you differentiate implicitly?

- Take the derivatives of both sides with respect to x.
- Collect all $\frac{dy}{dx}$ (or y') on one side.
- Solve for $\frac{dy}{dx}$ (or y').

Note: For the terms that contain y , we need to be careful. Remember that y is a rule that depends on x, but we do not know the specific rule.

$$\frac{d}{dx}\left(x^2\right)=2x$$ **(as usual).**

$$\frac{d}{dx}\left(y^2\right)=2y\frac{dy}{dx}$$

$$\frac{d}{dx}\left(y^3\right)=3y^2\frac{dy}{dx}$$

$$\frac{d}{dx}\left(xy\right)\overset{\text{use product rule}}{=}1\cdot y+x\cdot\frac{dy}{dx}$$

$$\frac{d}{dx}\left(x^2y^3\right)\overset{\text{use product rule}}{=}2x\cdot y^3+x^2\cdot 3y^2\frac{dy}{dx}$$

$$\frac{d}{dx}\left(5x^3y^2\right)\overset{\text{use product rule}}{=}15x^2\cdot y^2+5x^3\cdot 2y\frac{dy}{dx}$$

$$\frac{d}{dx}\left(xy+y^2\right)\overset{\text{use product rule on the first term}}{=}\underbrace{1\cdot y+x\cdot\frac{dy}{dx}}_{(xy)'}+\underbrace{2y\frac{dy}{dx}}_{(y^2)'}$$

$4x \cdot y$

Example: Find $\frac{dy}{dx}$ if $x^3 + y^2 + 4xy = 6$.

Implicit differentiation

take derivatives of both sides

$$3x^2 + 2 \cdot y \cdot \frac{dy}{dx} + \left(4 \cdot y + 4x \cdot \frac{dy}{dx}\right) = 0$$

$$(2y + 4x) \cdot \frac{dy}{dx} = -3x^2 - 4y$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{-3x^2 - 4y}{2y + 4x}}$$

$$\boxed{} = \boxed{}$$

Example: Given $4x^2 + xy^3 - 2y = 16$, find the slope of the tangent line at the point $(2,1)$.

$$\left.\frac{dy}{dx}\right|_{(2,1)} = ?$$

Implicit Differentiation

Take deriv. of both sides:

deriv. of 16

$$8x + (1 \cdot y^3 + x \cdot 3y^2 \cdot \frac{dy}{dx}) - 2 \cdot \frac{dy}{dx} = 0$$

$$(3xy^2 - 2) \cdot \frac{dy}{dx} = -8x - y^3$$

$$\frac{dy}{dx} = \frac{-8x - y^3}{3xy^2 - 2}$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{(2,1)} = \frac{-8\cdot 2 - 1^3}{3\cdot 2\cdot 1 - 2}$$

$x = 2$
$y = 1$

$$= \boxed{\frac{-17}{4}}$$

what if?

$(2,1)$ $m = -\frac{17}{4}$ equation

$$y - 1 = -\frac{17}{4}(x - 2)$$

$$\underbrace{3x + y} + \underbrace{5xy \cdot A} - \underbrace{2 \cdot A} = 0$$

$$\begin{cases} 5xy\,\underline{\underline{A}} - 2\underline{\underline{A}} = -3x - y \\ \underline{\underline{(5xy - 2)}} \cdot A = -3x - y \end{cases}$$

$$A = \frac{-3x - y}{5xy - 2}$$

$$A = \boxed{}$$

$$4x^2 + xy^3 - 2y = 16$$

$$8x + (1 \cdot y^3 + x \cdot 3 \cdot y^2 \cdot y') - 2y' = 0$$

Solve for y'

$$3xy^2 \cdot y' - 2 \cdot y' = -8x - y^3$$

$$\boxed{(3xy^2 - 2)} \cdot y' = -8x - y^3$$

$$y' = \frac{-8x - y^3}{3xy^2 - 2}$$

$$\stackrel{?}{} \quad 2^2 + 2\cdot 1 + 1^2 = 7$$
$$7 = 7 \checkmark$$

$x \cdot y$ product

Example: Given $x^2 + xy + y^2 = 7$,

a) Find the equation of the tangent line to the curve at the point $(2,1)$.

Implicit diff.
Take deriv. of both sides:

$$2x + (1\cdot y + x\cdot y') + 2\cdot y\cdot y' = 0$$

$$\Rightarrow \quad x\cdot y' + 2y\cdot y' = -2x - y$$

$$\boxed{(x+2y)}\cdot y' = -2x - y$$

$$y' = \frac{-2x-y}{x+2y}$$

$$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{-2\cdot 2 - 1}{2 + 2\cdot 1} = \frac{-5}{4} = m_{tan}$$

b) Find the point(s) where the curve has a horizontal tangent line.

$$\boxed{y - 1 = \frac{-5}{4}(x-2)}$$

$$\frac{dy}{dx} = \frac{-2x-y}{x+2y} \overset{\text{set}}{=} 0$$

$$\Rightarrow \quad \text{top} = 0 \quad (\& \text{ bottom} \neq 0)$$

$$-2x - y = 0 \Rightarrow y = -2x$$

$$x^2 + xy + y^2 = 7$$
$$x^2 + x(-2x) + (-2x)^2 = 7$$

$$\Rightarrow \underbrace{x^2}\ - \underbrace{2x^2} + \underbrace{4x^2} = 7$$

$$3x^2 = 7$$

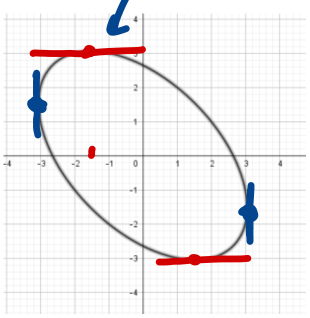
$$x^2 = \frac{7}{3} \Rightarrow x = \pm\sqrt{\frac{7}{3}}$$

$$\Rightarrow y = -2x$$

Points: $\left(\sqrt{\frac{7}{3}}, -2\sqrt{\frac{7}{3}}\right)$

$\left(-\sqrt{\frac{7}{3}}, 2\sqrt{\frac{7}{3}}\right)$

Check: $x^2+xy+y^2=7$



$\pm\sqrt{\frac{7}{3}}$

vertical tangent line

$$\frac{dy}{dx}=\frac{-2x-y}{x+2y}$$

bottom $=0$ (& top $\neq 0$)

Example: Find $\left.\frac{dy}{dx}\right|_{x=0}$ if $\cos(xy)=6x+2y$.

$\Leftarrow$ $\cos(0)=6\cdot 0+2y$

$1=2y \Rightarrow \underline{\underline{y=\frac{1}{2}}}$

Implicit differentiation:

$$-\sin(xy)\cdot(1\cdot y+x\cdot y')=6+2\cdot y'$$

$$-y\sin(xy)-x\cdot\sin(xy)\cdot y'=6+2y'$$

$$-x\sin(xy)\cdot\underline{\underline{y'}}-2\underline{\underline{y'}}=6+y\sin(xy)$$

$$y'\left[-x\sin(xy)-2\right]=6+y\sin(xy)$$

$$\frac{dy}{dx}=y'=\frac{6+y\sin(xy)}{-x\sin(xy)-2}$$

$$\left.\frac{dy}{dx}\right|_{x=0}=\frac{6}{-2}=\boxed{-3}$$

Example: Given $\underbrace{x^2 - y^2}_{} = 16$, find $\underline{\underline{\dfrac{d^2 y}{dx^2}}}$.

implicit diff.

$$2x - 2 \cdot y \cdot y' = 0 \Rightarrow -\cancel{2} y y' = -\cancel{2} x$$

$$y' = \frac{x}{y}$$

substitute

$$\frac{dy}{dx} = \frac{x}{y}$$

deriv.

$$\frac{d^2y}{dx^2} = \frac{1 \cdot y - x \cdot y'}{y^2} = \frac{y - x \cdot \frac{x}{y}}{y^2}$$

$$= \frac{y - \frac{x^2}{y}}{y^2} = \ldots$$

Exercise: What if one of the coordinates is not given?

Given $y + \sin(xy) = 4$, find the slope of the tangent line to the curve at x=0.

Exercise: Given $2xy + y^3 + x^2 = 4$, find the equation of the tangent line when $x = 1$.

Exercise: Given $2\sin(x)\cdot\cos(y) = 1$, find $\dfrac{dy}{dx}$.