completed notes

Math 2413- Calculus I

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- Check CASA calendar for due dates.
- Bring "blank notes" to class. Completed notes will be posted after class.
- Do your best to attend every lecture and lab.
- Study after every lecture; work on the quiz covering the topic we cover on the lecture immediately afterwards. Retake your quizzes for more practice.
- Get help when you need help; bring your questions to the labs, or my office hours. We also have tutoring options on campus.
- Respect your friends in class; stay away from distractive behavior. Do your best to concentrate on the lecture.
- If you email me, mention the course code in the subject line. Email is the best way to communicate with me outside of class. Teams chat messages are not monitored or replied to.

section 3.1: finished on 2/16

Today (Day 12): 3.2, 3.3, maybe 3.4?

EHW 1: S 3.1 & 3.2 (due on Tuesday)

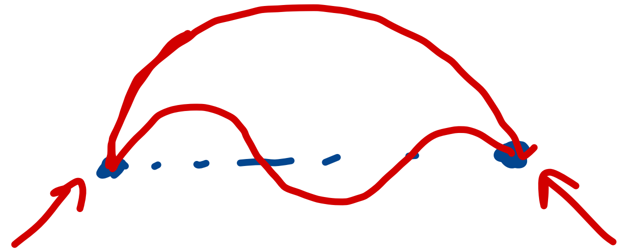
- Supplemental Video about related rates posted on CASA.

Math 2413

Chapter 3

Applications of the Derivative

Chapter 3- Applications of Derivatives



Section 3.2 Mean Value Theorem

In this section, we discuss 2 important theorems that apply under certain conditions.

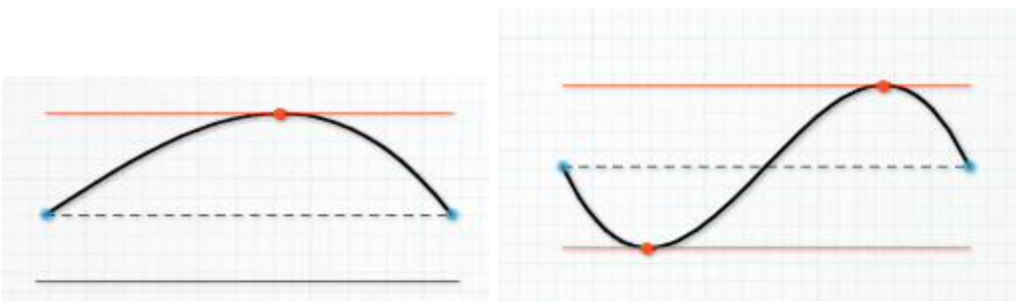
Theorem: Rolle's Theorem

Suppose that f is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b).

open interval

If $f(a)=f(b)$, then there is at least one number c in (a,b) for which $f'(c)=0$.

The essence of Rolle's theorem may be seen on these pictures:



Rolle's theorem is sometimes stated as follows:

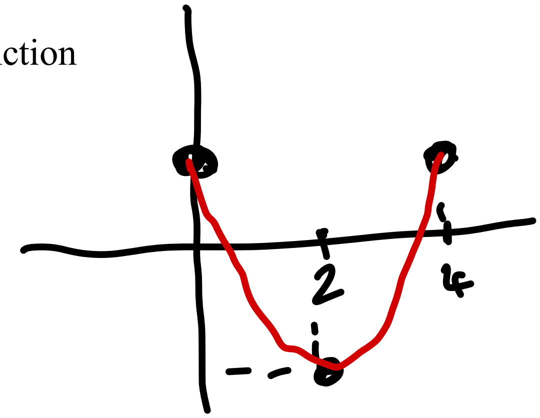
Suppose that f is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b). If $f(a)=f(b)=0$, then there is at least one number c in (a,b) for which $f'(c)=0$.

That is, Rolle's theorem tells us that between any two roots of f, there must be a root of f'.

Example: Verify that the Rolle's Theorem applies to the function

$$f(x) = x^2 - 4x + 2, \quad [0,4]$$

Find all points in this interval that satisfy Rolle's Theorem.



Conditions of Rolle's theorem to be checked:

(1) Is the function continuous on [0,4]? ✓
(2) Is the function differentiable on (0,4)? ✓
(3) Is $f(0)$ equal to $f(4)$?

$f(0) = 0 - 4 \cdot 0 + 2 = 2$
$f(4) = 4^2 - 4 \cdot 4 + 2 = 2$ $\Big\} =$ ✓

Rolle's theorem applies.

There is at least one c in (0,4) where $f'(c) = 0$.

To find these values:

$f'(x) = 2x - 4 \stackrel{?}{=} 0$ solve!

$2x = 4 \Rightarrow x = 2$ in (0,4) ✓

$\boxed{c=2}$

Example: Verify that the Rolle's Theorem applies to the function

$$f(x)=\cos(2x),\ \boxed{[0,\pi]}$$

inside $(0,\pi)$

Find all points in this interval that satisfy Rolle's Theorem.

Conditions of Rolle's theorem to be checked:

(1) Is the function continuous on the given interval? ✓
(2) Is the function differentiable on the given interval? ✓
(3) Is $f(0)$ equal to $f(\pi)$?

$f(0) = \cos(2\cdot 0) = 1$
$f(\pi) = \cos(2\cdot\pi) = 1$
$=$ ✓

Rolle's theorem applies.
There is at least one c in $(0,\pi)$
where $f'(c)=0$.

where? $f'(x) = 0$ & solve.

$-2.\sin(2x) = 0$

$\Rightarrow \sin(2x) = 0$ in $(0,\pi)$

$2x=0$, $2x=\pi$, $2x=2\pi$, $2x=3\pi$, ...

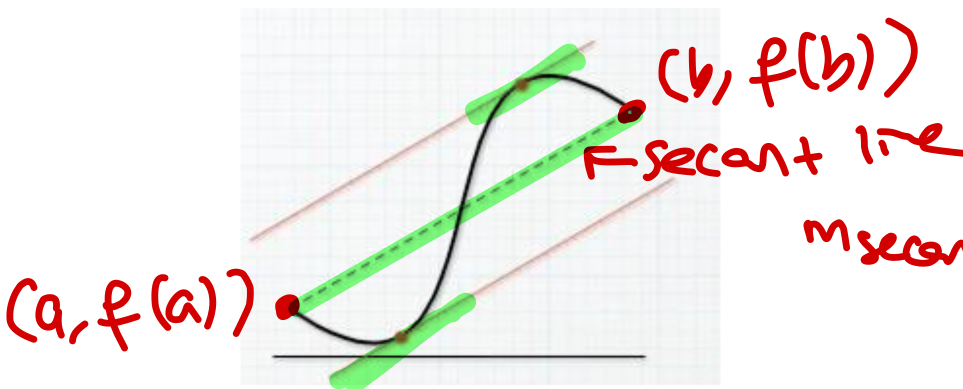
$\cancel{x=0}$ $c \to \boxed{x=\frac{\pi}{2}}$ $\cancel{x=\pi}$ bigger...

The mean-value theorem is a generalization of the Rolle's Theorem.

Theorem: The Mean-Value Theorem

Suppose that f is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b). There is at least one number c in (a,b) for which

$$f'(c)=\frac{f(b)-f(a)}{b-a}.$$



$$m_{secant} = \boxed{\frac{f(b)-f(a)}{b-a}}$$

Of course, there may be more than one point where the tangent line is parallel to the secant line.

See this visual (geogebra file); move the sliders:

https://www.geogebra.org/m/qTneMSa4

Example: Verify that the Mean Value Theorem applies to the function

$$f(x) = x^3 - 2x + 1, \quad [0,2]$$

Find all points in this interval that satisfy Mean Value Theorem.

$$m_{sec} = \frac{f(2) - f(0)}{2-0}$$

Conditions to check for MVT:

(1) Is this function continuous on [0,2]? ✓

(2) Is this function differentiable on (0,2)? ✓

$$m_{secant} = \frac{5-1}{2} = \boxed{2}$$

Hence, MVT applies.

By MVT, there is at least one c in (0,2) such that $f'(c) = \boxed{2}$

Where?

Set $f'(x) = 2$ & solve.

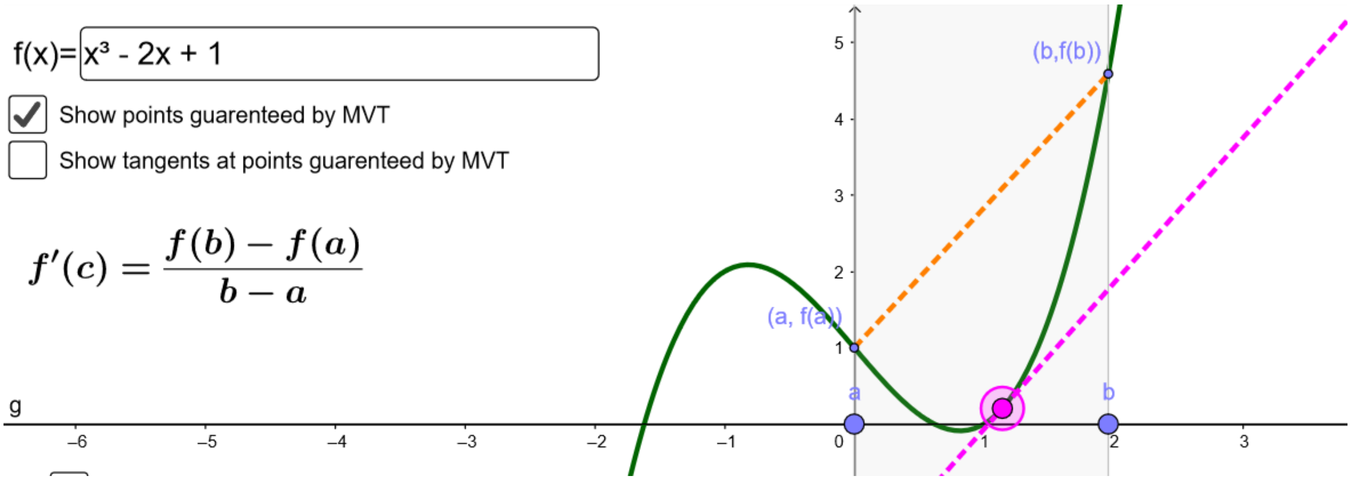
$$3x^2 - 2 = 2$$

$$\Rightarrow 3x^2 = 4$$

$$\Rightarrow x^2 = \frac{4}{3} \Rightarrow x = \boxed{\pm\sqrt{\frac{4}{3}}} \; ?$$

c inside (0,2): $c = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

Here's the picture:



$f'(x) = 1 - \frac{1}{x^2}$

Example: Verify that the Mean Value Theorem applies to the function

$f(x) = x + \frac{1}{x}, \quad \boxed{[1,4]}$

$f(4) = \underbrace{4 + \frac{1}{4}}$ $\quad f(1) = 1 + 1 = 2$

Find all points in this interval that satisfy Mean Value Theorem.

$m_{secant} = \frac{9}{12}$!

Conditions to check for MVT:

(1) Is this function continuous on [1,4]? ✓
(2) Is this function differentiable on (1,4)? ✓

Hence, MVT applies.

By MVT, there is at least one c in (1,4)

where $f'(c) = \frac{f(4) - f(1)}{4 - 1} = \frac{\frac{17}{4} - 2}{3}$

$f'(c) = \frac{\frac{9}{4}}{3} = \frac{9}{12}$

where? $\quad f'(x) = 1 - \frac{1}{x^2} = \frac{9}{12}$ & solve.

Example: Does the Mean Value Theorem apply to the function over the given interval?

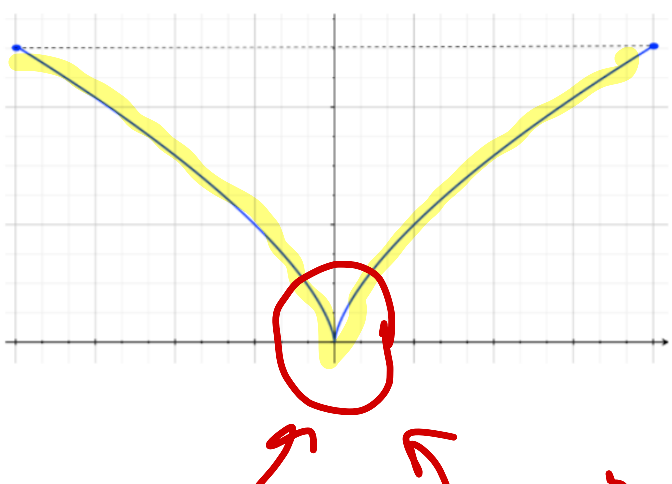
$$f(x)=\frac{x+2}{x}, \quad [-1,2]$$

No

f(x) is NOT continuous on [-1,2].

Example: Does the Mean Value Theorem apply to the function over the given interval?

$$f(x)=x^{2/3}, \quad [-1,1]$$



1) cts? ✓

2) differentiable? ✗

Not diff.

MVT does not apply.

Exercise: Verify that the Mean Value Theorem applies to the function

$$f(x)=\sqrt{4-x^2}, \quad [0,2]$$

Find all points in this interval that satisfy Mean Value Theorem.

Conditions to check for MVT:

(3) Is this function continuous on [0,2]? ✓

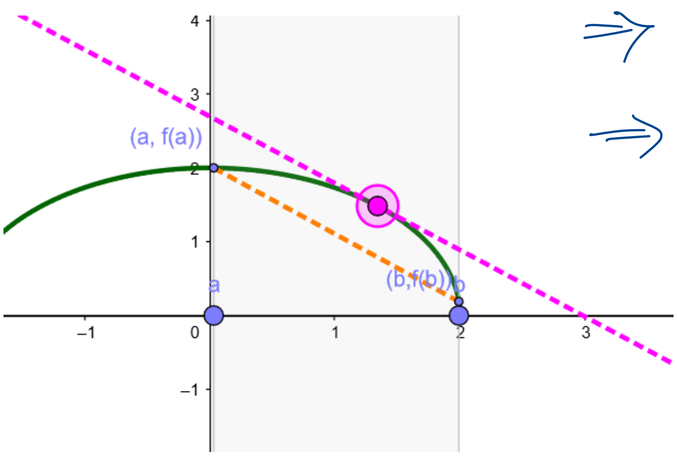
(4) Is this function differentiable on (0,2)? ✓

Hence, MVT applies.

By MVT, there is at least one c in $(0,2)$ where $f'(c) = \frac{f(2)-f(0)}{2-0} = \frac{0-\sqrt{4}}{2} = 1$

where? $f'(x) = \frac{-2x}{2\sqrt{4-x^2}} = 1$

Check:



$\Rightarrow \sqrt{4-x^2} = -x$ (square both sides)

$\Rightarrow 4-x^2 = x^2$

$\Rightarrow 2x^2 = 4$

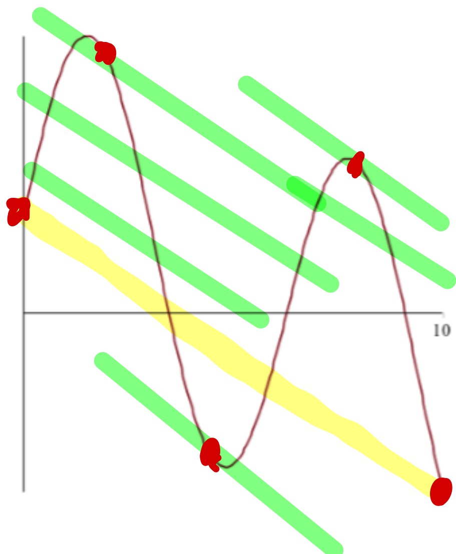
$\Rightarrow x^2 = 2$

$\Rightarrow x = \pm\sqrt{2}$

Pick $\boxed{c=\sqrt{2}}$ since it has to be in $(0,2)$

Online quiz & EHW1

Exercise: At how many points between 0 and 10 does the function satisfy the Mean Value theorem?



Exercise: Does Rolle's Theorem apply to the given function over the indicated interval? Does Mean Value Theorem apply? If a theorem applies, find the values of c in the given interval that satisfy the theorem.

	Rolle's Theorem?	MVT?	Value(s) of c
$f(x)=x^3-12x$, [0,3]			
$f(x)=cos(2x)$, $[0,\pi]$			
$f(x)=2\sqrt{x}-x$, [0,4]			
$f(x)=2\sqrt{x}-x$, [0,1]			
$f(x)=2x^{1/3}$, [-1,1]			
$f(x)=2x^{1/3}$, [1,8]			
$f(x)=\frac{x^2}{x-2}$, [1,3]			
$f(x)=\frac{x^2}{x-2}$, [0,1]			

Exercise: Verify that the Mean Value Theorem applies to the function

$$f(x)=\sin(4x), \quad \left[0,\frac{\pi}{2}\right]$$

Find all points in this interval that satisfy Mean Value Theorem.