Completed

See Extra sign chart examples at the end

Math 2413- Calculus I

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- Check CASA calendar for due dates.
- Bring "blank notes" to class. Completed notes will be posted after class.
- Do your best to attend every lecture and lab.
- Study after every lecture; work on the quiz covering the topic we cover on the lecture immediately afterwards. Retake your quizzes for more practice.
- Get help when you need help; bring your questions to the labs, or my office hours. We also have tutoring options on campus.
- Respect your friends in class; stay away from distractive behavior. Do your best to concentrate on the lecture.
- If you email me, mention the course code in the subject line. Email is the best way to communicate with me outside of class. Teams chat messages are not monitored or replied to.

Remark:
To answer most questions in this section, you will need to solve inequalities.
You might want to review solving inequalities before working on this section;

Watch this video:
https://online.math.uh.edu/courses/placement/3c.mp4

Exercise: Solve this inequality: $\frac{-2x(x-1)^3(x+2)}{(x-4)^2} \geq 0$

Exercise: Solve this inequality: $4x(x^2-1)(x^2+2x+1) < 0$

practice!

Solving inequalities using a sign chart.

Section 3.3 – Intervals of Increase and Decrease and Extreme Values

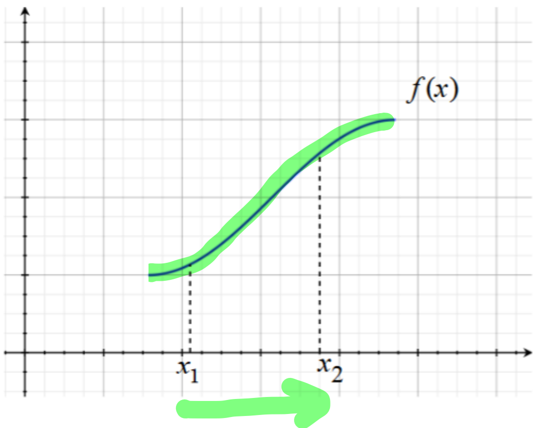
Definition: Let f be a function whose domain includes an interval I.

We say that f is **increasing** on I if for every two numbers x_1, x_2 in I,

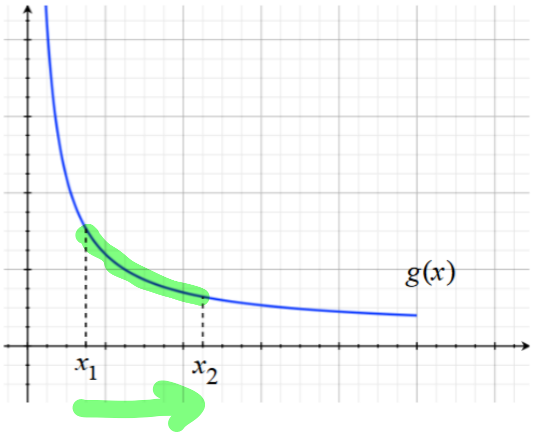
$$x_1 < x_2 \text{ implies that } f(x_1) < f(x_2).$$

We say that f is **decreasing** on I if for every two numbers x_1, x_2 in I,

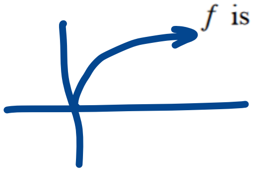
$$x_1 < x_2 \text{ implies that } f(x_1) > f(x_2).$$

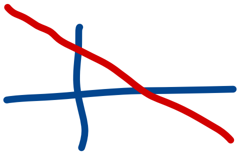


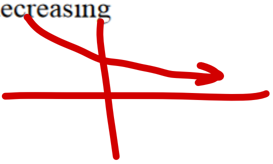
f is increasing



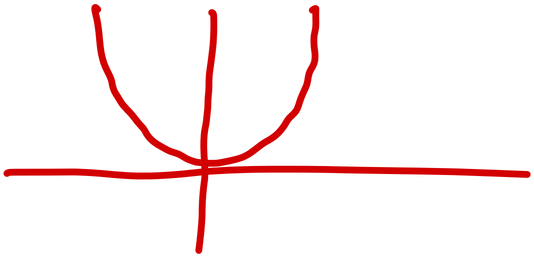
g is decreasing

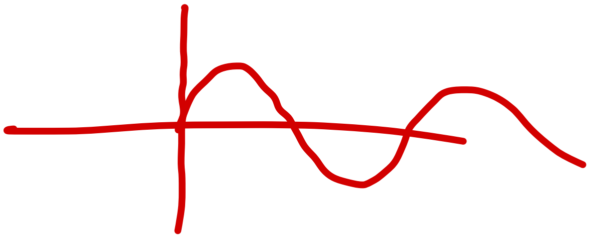




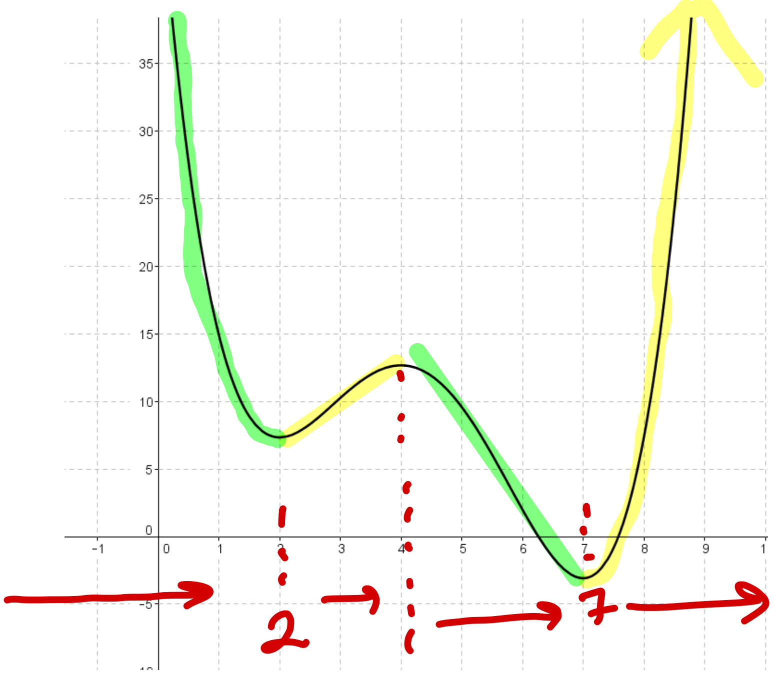


If the graph of a function is given, it is very easy to find the intervals of increase and decrease. Simply observe whether the y–values are going up or down.





Example: The graph of $f(x)$ is given below:

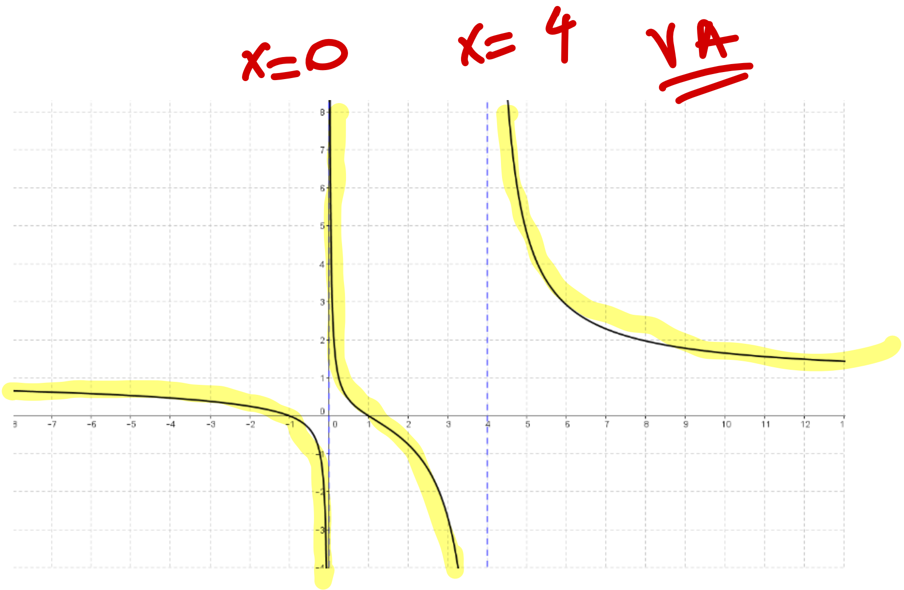


The function is increasing over the intervals:

$(2,4)$ and $(7,\infty)$

The function f is decreasing over the intervals:

$(-\infty,2)$ and $(4,7)$

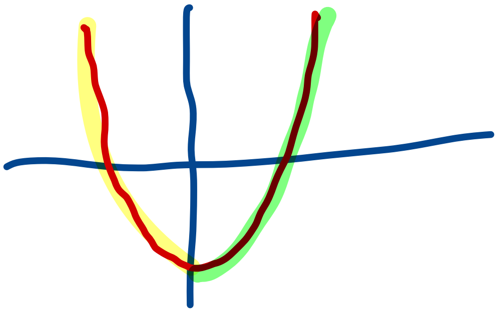


The function f is increasing over the intervals: X Never

The function f is decreasing over the intervals:

$(-\infty,0)$ and $(0,4)$ and $(4,\infty)$

Can NOT say ~~$(-\infty,\infty)$~~



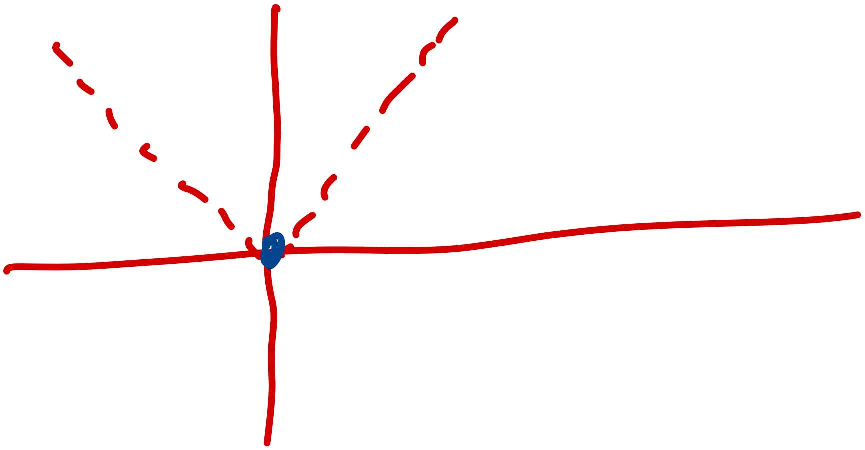
What if the function is given by a formula?

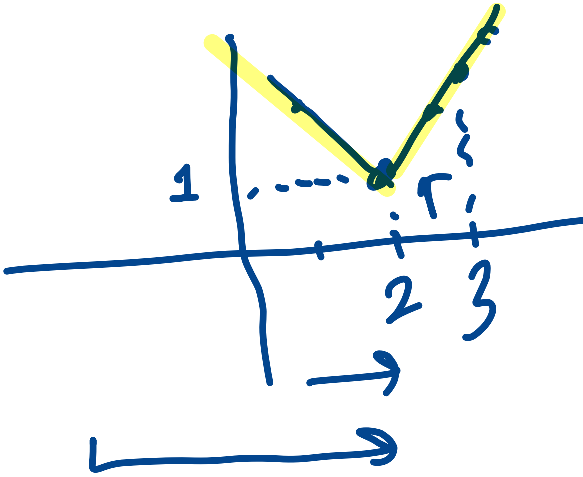
$f(x)=x^2-1$ or $g(x)=\sin(x)$?

Option 1: If you know how to graph the function, graph and observe the behavior.

$5\cdot 0+1$

Example: Given $f(x)=5|x-2|+1$, when is this function increasing? When is it decreasing?



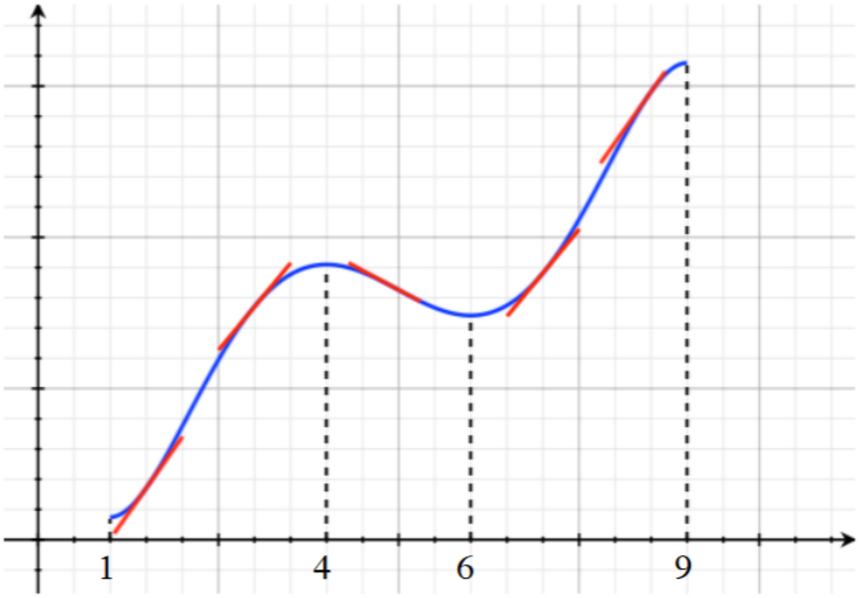


$(-\infty, 2)$: decreasing

$[2, \infty)$: increasing

What if?? $f(x)=x^4-5x^3+x-2$ or $f(x)=x^2\cos(x)$? What if we don't know how to graph the function?

Let's look at a graph to observe the relation between the slope of the tangent line, and the fact that the function is increasing or decreasing.



Observe that over the intervals where the function is increasing, the tangent lines have positive slope. On the other hand, over the intervals of decrease, the tangent lines have negative slope.

Geogebra file – tangent line; increasing/decreasing functions:

https://www.geogebra.org/m/BnfjWrB8

Theorem:

Suppose that f is differentiable on the interior of an interval I and continuous on all of I.

If $f'(x) > 0$ for all x in I, then f increases on I.
If $f'(x) < 0$ for all x in I, then f decreases on I.

Remark: Conversely, we can say the following:

If f increases on I, then $f'(x) \geq 0$ for all x in the interior of I.
If f decreases on I, then $f'(x) \leq 0$ for all x in the interior of I.

Example: Given $f(x) = x^2 - 10x + 2$, when is this function increasing? When is it decreasing?

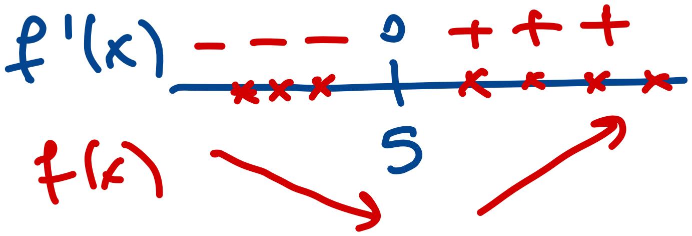
$$f'(x) = 2x - 10 = 2\underbrace{(x-5)}$$

> 0?

< 0?

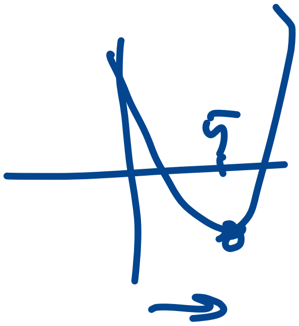
$\boxed{x=5}$ is a zero

sign chart



increasing on: $(5, \infty)$

decreasing on: $(-\infty, 5)$



Example: Given $f(x)=6x^5-40x^3+10$, when is this function increasing? When is it decreasing?

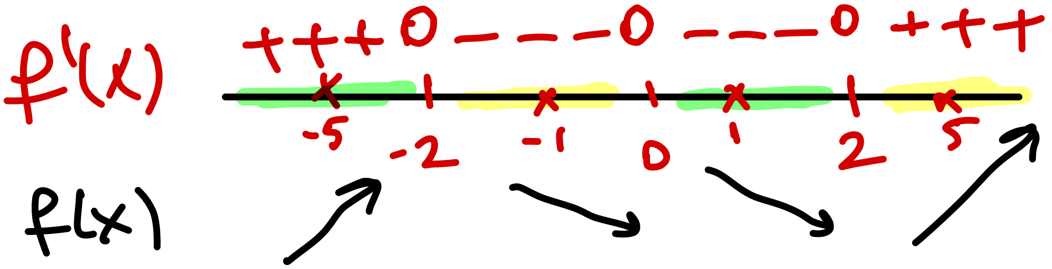
$f'(x) = 30.x^4 - 120x^2 = 0$?

$30x^2(x^2-4) = 0$

$\rightarrow$ $\boxed{30\underline{x^2}\,\underbrace{(x-2)}\cdot\underbrace{(x+2)}} = 0$

zeros: $x=0$, $x=2$, $x=-2$

sign chart:



f is increasing on: $(-\infty, -2)$ & $(2, \infty)$

f is decreasing on: $(-2, 0)$ & $(0, 2)$

combine? ✓

$(-2, 2)$ ✓

Question:

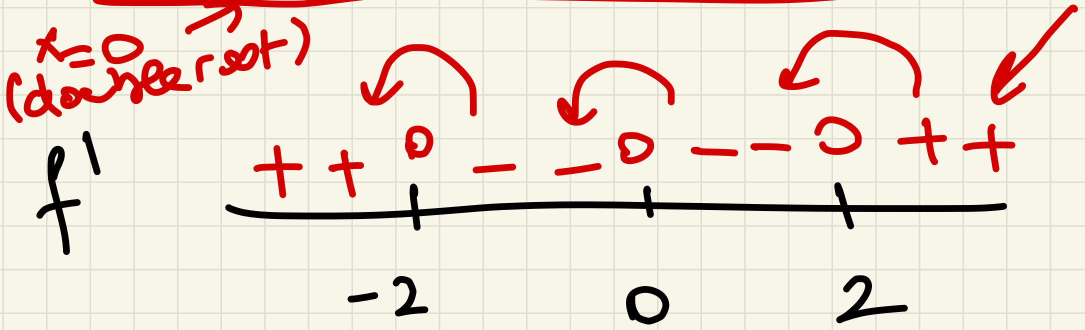
can combine if polynomial (defined)

optional way

odd odd

$$\boxed{30\boxed{x^2}(x-2)(x+2)}$$

x=0 (double root)



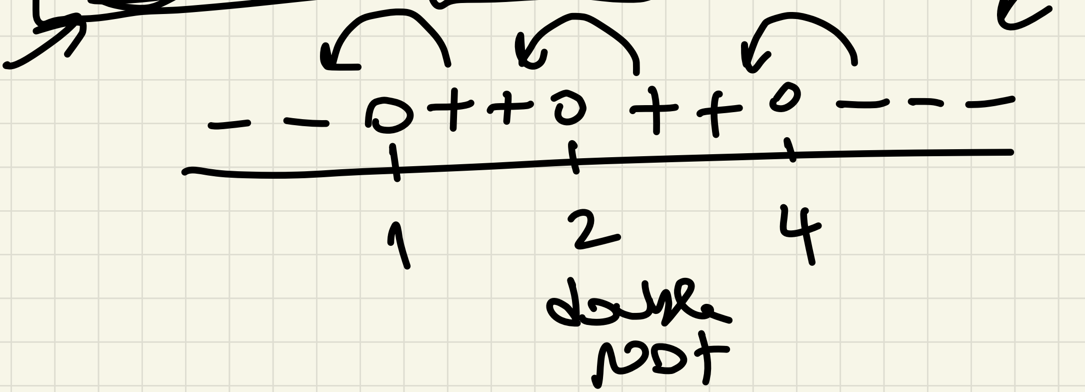
OR

even power

$= -5 \cdot + \cdot + \cdot +$

even

$$\boxed{-5}(x-1)^1(x-2)^2(x-4)^{\boxed{3}}$$



even

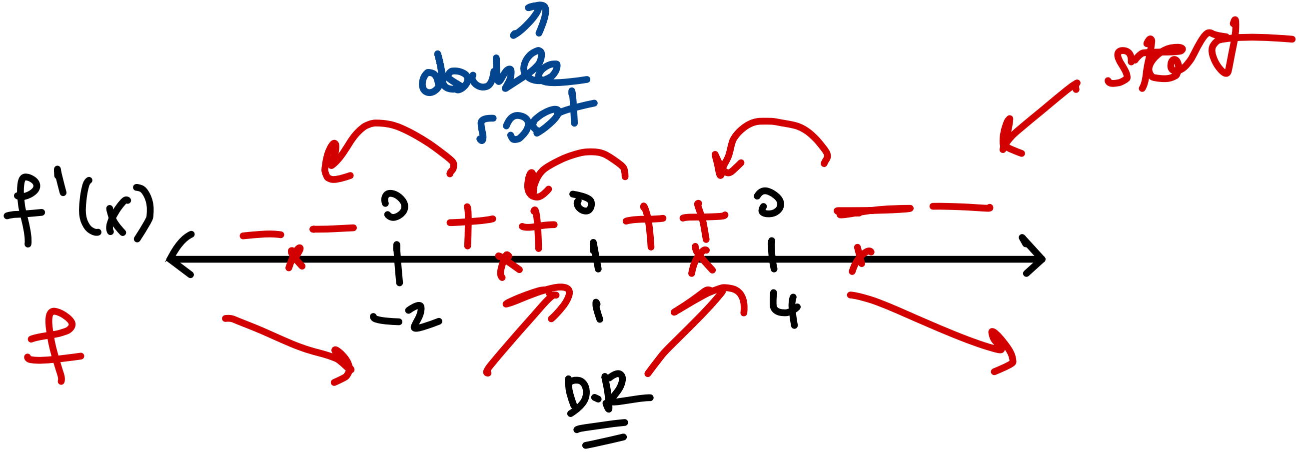
$-2 \cdot + \cdot + \cdot +$

Example: Given $f'(x) = \boxed{\underbrace{-2}\,\overset{\text{even}}{(x-1)^2}(x+2)^3(x-4)}$ (be careful; derivative is given!), when is the function $f(x)$ increasing? When is it decreasing?

sign chart for f'

zeros of f' : $\underbrace{x=1}$, $x=-2$, $x=4$

double root



f is increasing on: $(-2,1)$ & $(1,4)$

combine? Yes $(-2,4)$

f is decreasing on: $(-\infty,-2)$ & $(4,\infty)$

$f'(x) = A \cdot B \cdot C \cdot D$

+	+	+	+
+	−	+	+
−	−	−	−

$(\qquad)(\qquad)(\qquad)(\qquad)$

$(x-2)^3$

$\underbrace{(x-9)^{\boxed{2}}}_{+}$

Example: Given $f(x)=\dfrac{x^2}{x-4}$, when is this function increasing?

VA: $x=4$

Solution:

Step 1: Take the derivative of this function (using quotient rule):

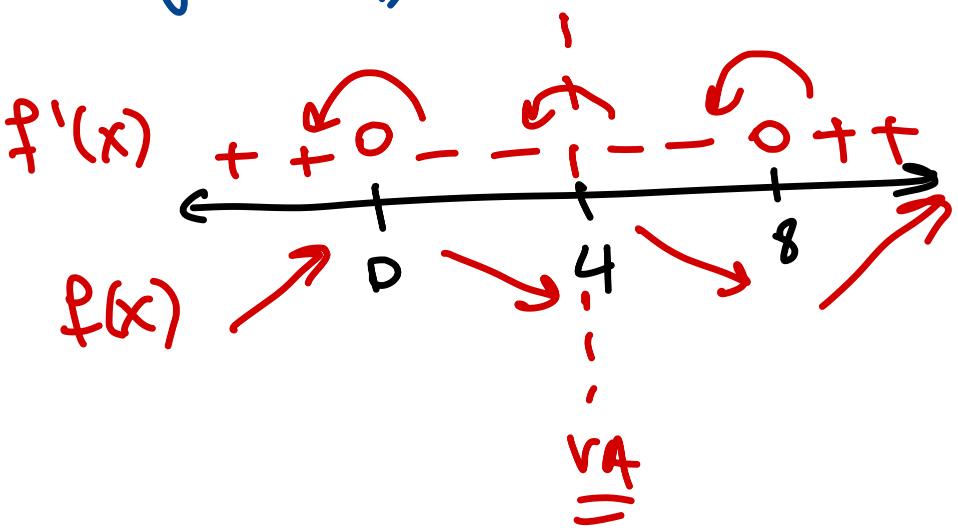
$$f'(x)=\frac{x^2-8x}{(x-4)^2}$$

Step 2: Need to solve: $f'(x)=\dfrac{x^2-8x}{(x-4)^2}=\dfrac{x(x-8)}{(x-4)^2}>0$. Make a sign chart.

even power

$f'=0$? top $=0$ if $x=0$, $x=8$

f': undefined if denom$=0$; $x=4$ (V.A)



f is increasing on: $(-\infty,0)$ & $(8,\infty)$

decreasing on: $(0,4)$ & $(4,8)$

can NOT combine.

S 3.3 Attendance Day 13

Q#① $f(x) = x^3 - 3x+1$

When is f(x) decreasing? = ↘ if $f' < 0$

a) $(1, \infty)$

[✓ b) $(-1, 1)$]

c) $(-\infty, -1)$ and $(1, \infty)$

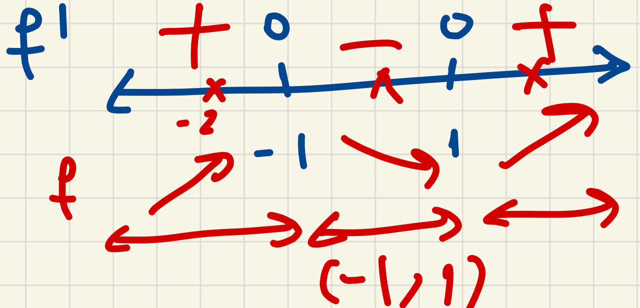
d) $(-\infty, -1)$

e) None of the above

use a sign chart for $\underline{f'(x)}$

$$f'(x) = 3x^2-3 = 3\underbrace{(x-1)}\underbrace{(x+1)}$$

−	−
−	+
+	+



§ 3.3

* Practice using sign charts to solve inequalities!!!

* EHW 1 is due tonight (covers 3.1 & 3.2)

* Labquiz on Thursday

* Answersheet will be given back in lab on Thursday.

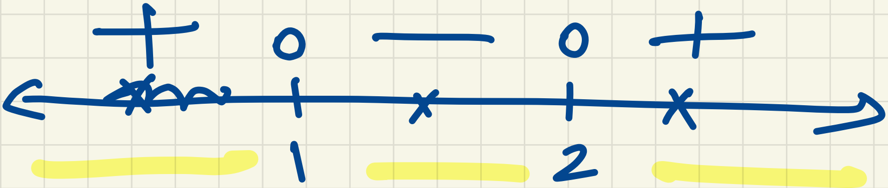
* $\underbrace{\text{Test 1}}_{\text{out of } 76} + \underbrace{\text{Test 1 FR}}_{\text{out of } 24} + \underbrace{5\% \text{ PT1}}_{\text{max: } 5}$

all raw scores!

* On EHW1

F - None of the above if needed

$$M = \underbrace{\overbrace{(x-1)}^{\substack{+\\+}} \cdot \overbrace{(x-2)}^{\substack{+\\-}}} \quad \substack{>0\\ \\<0}$$



$M > 0$ if x in $(-\infty, 1)$
& $(2, \infty)$

$M < 0$ if x in
$(1, 2)$

✓ Rational Functions

Example: Given $f(x)=\frac{2x}{x^2-1}$, when is this function increasing? When is it decreasing?

$\frac{\quad}{(x-1)(x+1)}$

VA: $x=1$, $x=-1$

Solution:

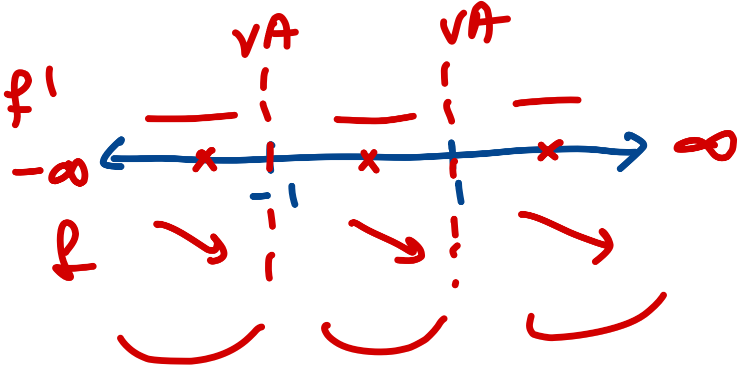
Step 1: Take the derivative of this function (using quotient rule):

$$\rightarrow f'(x)=\frac{-2(x^2+1)}{(x^2-1)^2}=\frac{\boxed{-2}\overset{+}{\boxed{(x^2+1)}}}{\underbrace{(x-1)^2}_{+}\underbrace{(x+1)^2}_{+}} \leftarrow \quad f'(-2)=\frac{-2\cdot(+)}{(+)\cdot(+)}$$

Step 2: Need to study the sign of $f'(x)$; make sure it is factored completely, then make a sign chart.

$f'=0$? Never

f': undefined? $x=1$, $x=-1$ (VA)



can NOT combine

f is decreasing : $(-\infty,-1)$ and $(-1,1)$ and $(1,\infty)$

✓ $\sqrt[3]{(x-4)^2}$: Domain: $(-\infty,\infty)$

Example: Given $f(x)=(x-4)^{2/3}$, when is this function increasing? When is it decreasing?

Step 1: Find the derivative.

$$f'(x)= \frac{2}{3}(x-4)^{-1/3}\cdot(1) = \frac{\boxed{2}}{3(x-4)^{1/3}}$$

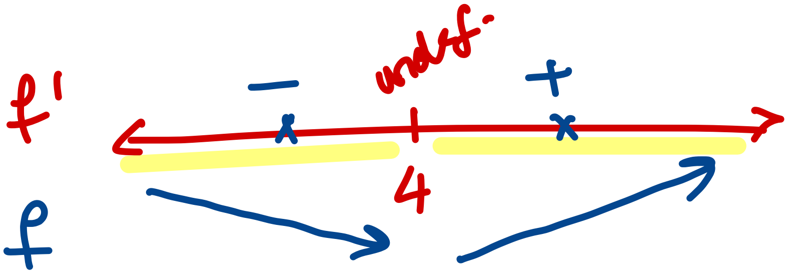
Step 2: Make a sign chart for the derivative function.

$$f'(3)=\frac{2}{3(-1)^{1/3}} < 0$$

$$f'(5)=\frac{2}{3\cdot 1^{1/3}}$$

$f'=0$? Never

f' : undefined : $x=4$



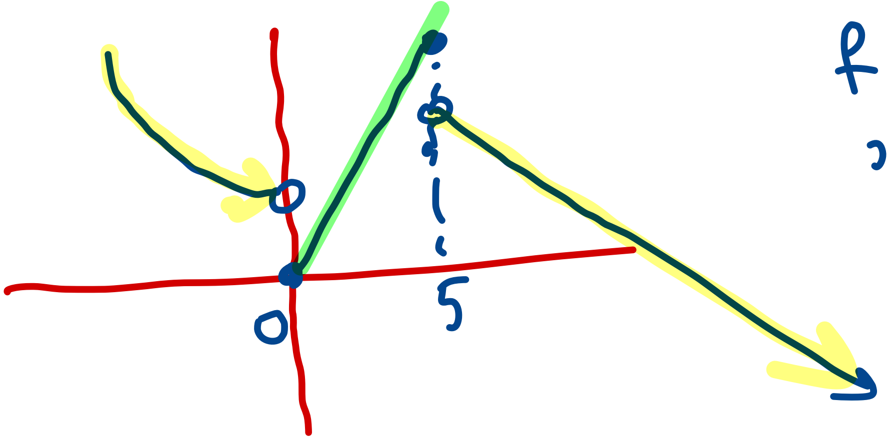
f is increasing on $(4,\infty)$

f is decreasing on $(-\infty,4)$

REMARK: In some cases, you may need to sketch the graph to answer questions about increasing/decreasing instead of working with the derivative. If the function is piecewise, use the graph. See your textbook for an example about how using just the derivative might give the wrong answer for piecewise functions (S3.3 of your textbook).

Piecewise function: Graph to answer!

Exercise: Given $f(x)=\begin{cases} x^2+1, & x<0 \\ 2x, & 0\le x\le 5 \\ -x, & 5<x \end{cases}$, when is this function increasing? When is it decreasing?

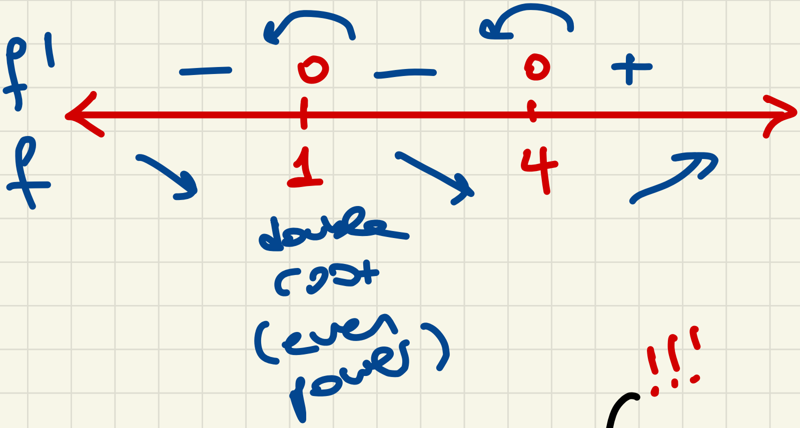


f is increasing on (0,5)

decreasing on (−∞,0) & (5,∞)

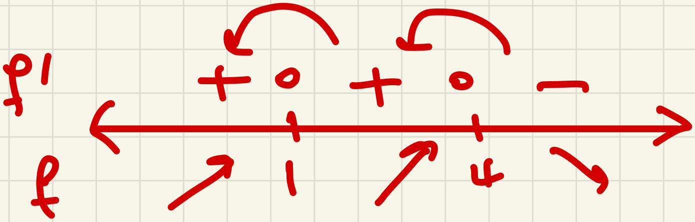
Additional Sign Chart Examples

Ex1 Say: $f'(x) = 5(x-1)^2(x-4)$



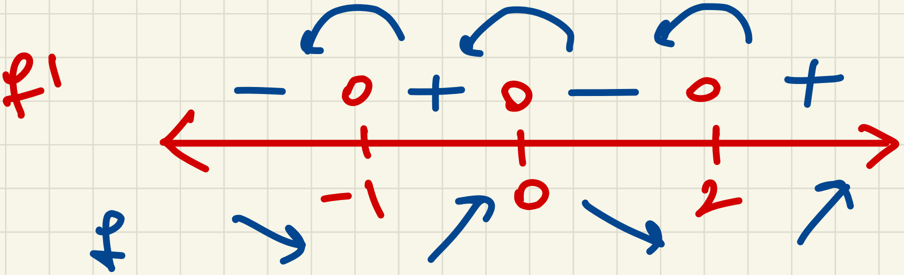
!!!

Ex2: Say: $f'(x) = -5 \cdot (x-1)^2 \cdot (x-4)$



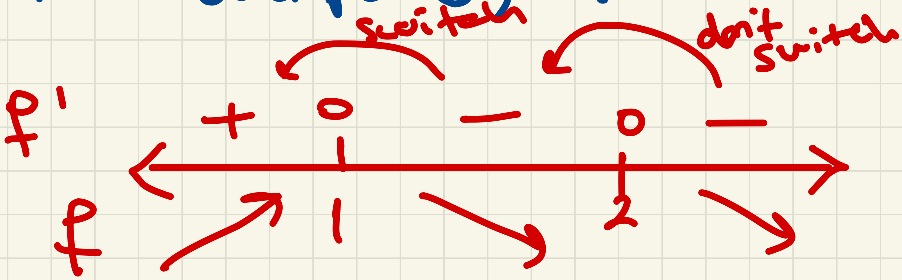
Ex3: Say $f'(x) = 2x \cdot (x+1)(x-2)^3$

zeros: $x=0$, $x=-1$, $x=2$
(all from odd powers)



Ex4: Say: $f'(x) = -2 \cdot (x-1)^3 (x-2)^2$

$x=1$ odd power, $x:2$ even power



f is increasing on: $(-\infty, 1)$