Math 2413- Calculus I

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- Check CASA calendar for due dates.
- Bring “blank notes” to class. Completed notes will be posted after class.
- Do your best to attend every lecture and lab.
- Study after every lecture; work on the quiz covering the topic we cover on the lecture immediately afterwards. Retake your quizzes for more practice.
- Get help when you need help; bring your questions to the labs, or my office hours. We also have tutoring options on campus.
- Respect your friends in class; stay away from distractive behavior. Do your best to concentrate on the lecture.
- If you email me, mention the course code in the subject line. Email is the best way to communicate with me outside of class. Teams chat messages are not monitored or replied to.

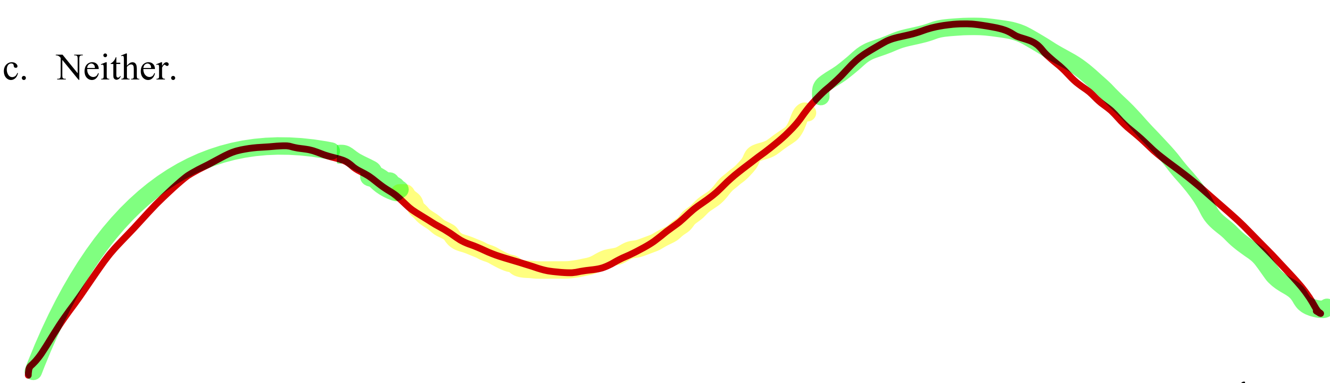
Solve before class!

Question# If $f''(x) = x^4 - 2x^3 - 1$, classify the critical point x=1 as local min/max.

a. f has local min at x=1.

b. f has local max at x=1.

c. Neither.

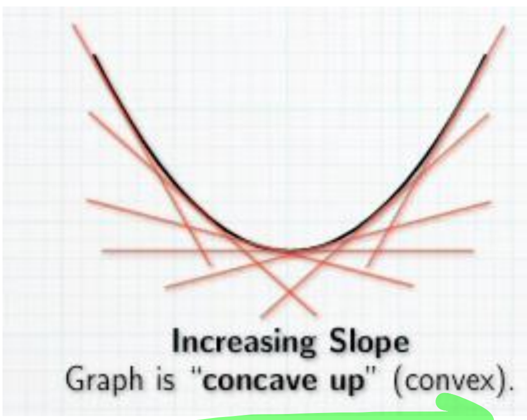


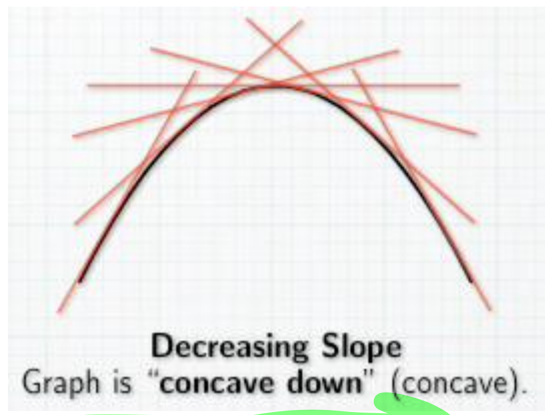
Section 3.5 – Concavity and Points of Inflection

Definition: Let f be a function that is differentiable on an open interval I.

The graph of f is **concave up** if f' is increasing on I.

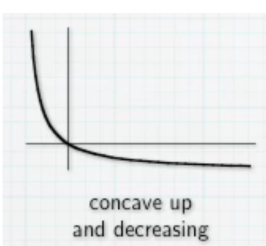
The graph of f is **concave down** if f' is decreasing on I.

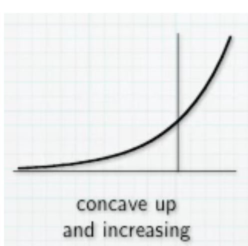


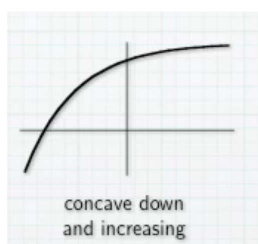


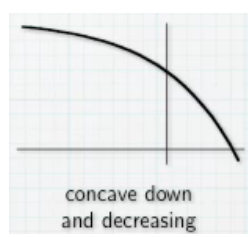
Even though both pictures indicate a local extreme value, note that that need not be the case.

Here are some graphs where the functions are concave up or down without any local extreme values.

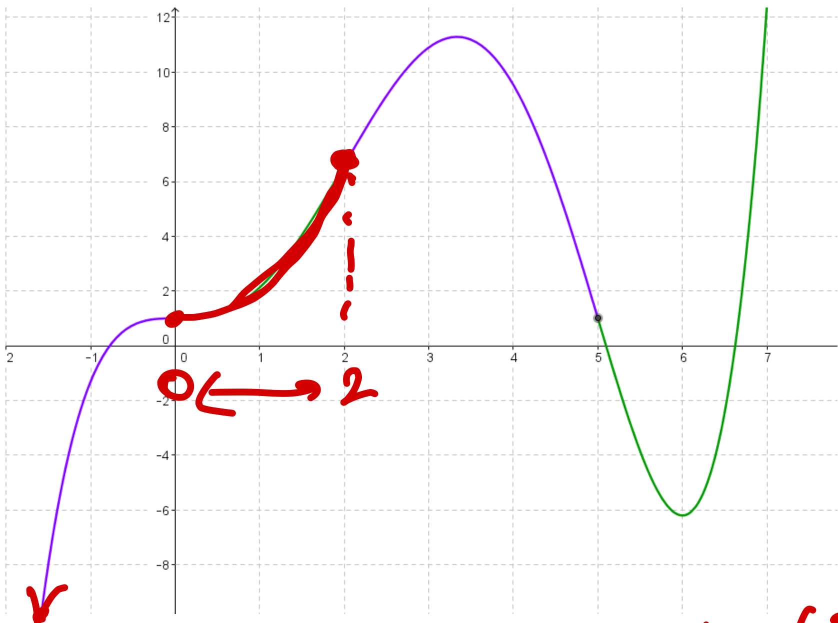








Example: The graph of a function f is given below:



When is f concave up? $(0,2)$ and $(5,\infty)$

When is f concave down? $(-\infty,0)$ & $(2,5)$

Points of inflection occur where the concavity changes.

Definition:

Let f be a function which is continuous at c and differentiable near c. The point $(c, f(c))$ is a point of inflection if the graph of f changes concavity at $x = c$.

$f \cup$ if $f' \nearrow$ if $f'' > 0$

Theorem: Let f be a function that is twice differentiable on an open interval I.

(i) If $f''(x) > 0$ for all x in I, then the graph of f is concave up on I.

(ii) If $f''(x) < 0$ for all x in I, then the graph of f is concave down on I.

$f''(x)$

The idea here is that if $f''(x) > 0$ for all x in I, then the derivative function f' is increasing, and hence by the definition of concavity, f concaves up on I.

How do we find the points of inflection?

FACT: If $(c, f(c))$ is a point of inflection, then either $f''(c) = 0$ or $f''(c)$ does not exist.

This gives us a way of finding possible inflection points; find the numbers where the second derivative is 0 or undefined, and investigate them to see whether or not concavity changes at those points.

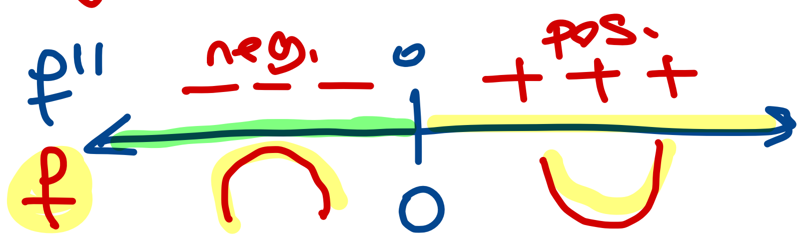
$x=0$ is a point of inflection
$(0, f(0))$ since concavity changes at $x=0$.

Example: Find the points of inflection and determine the concavity of $f(x)=x^3$.

$f'(x) = 3x^2$

$f''(x) = \boxed{6x} = 0 \Rightarrow \underline{x=0} \leftarrow$ to study

sign chart for f''



f is concave up on $(0, \infty)$

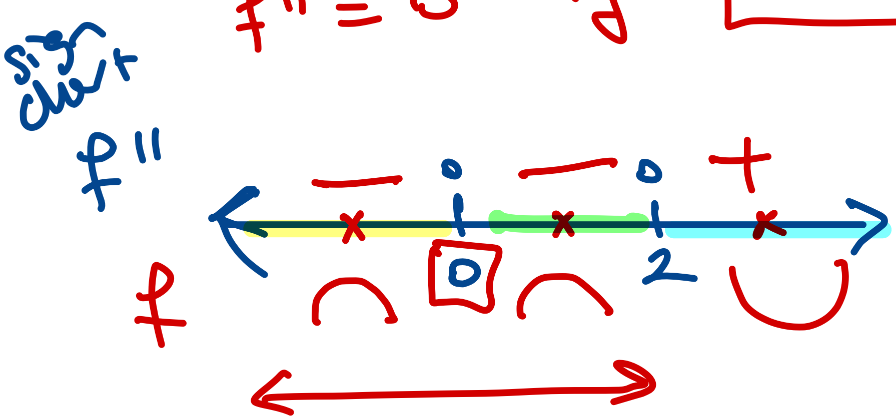
f is concave down on $(-\infty, 0)$

Example: Find the points of inflection and determine the concavity of $f(x)=6x^5-20x^4+2x$.

$f'(x) = 30x^4 - 80x^3 + 2$

$f''(x) = 120x^3 - 240x^2 = \boxed{120x^2(x-2)}$

$f''=0$ if $\boxed{x=0, \quad x=2}$



f is concave up on: $(2, \infty)$

f is concave down on: $(-\infty, 0)$ & $(0, 2)$

can combine: $\boxed{(-\infty, 2)}$

Point of inflection? at $x=2$; $(2, f(2))$

Concavity changes at $x=2$, so it is a PoI

Rational function

Example: Given $f(x)$, find points of inflection and the intervals where the graph is concave up and concave down.

$$\boxed{f(x)=\frac{x}{x^2-1}}$$

Domain: $x \neq 1,\ x \neq -1$

$$f'(x)=\frac{-\left(x^2+1\right)}{\left(x^2-1\right)^2}$$

$$\boxed{f''(x)=\frac{2x\left(x^2+3\right)}{\left(x^2-1\right)^3}}$$

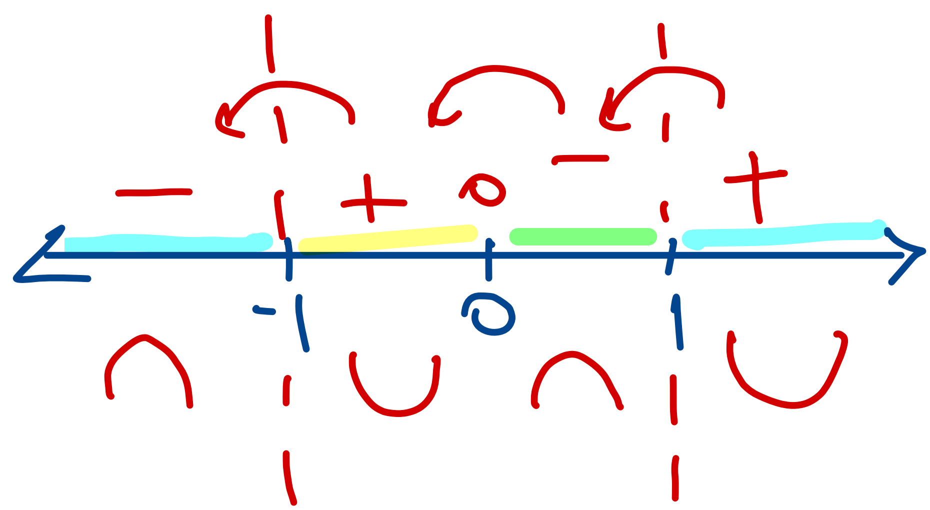
$(x-1)^3 \cdot (x+1)^3$

$f''=0$? if top $=0$ $\boxed{x=0}$

f'': undefined? if bottom $=0$

$x=1,\ x=-1.$

Sign chart for f'':



f

f is concave up on: $(-1,0)$ & $(1,\infty)$

f is concave down on: $(-\infty,-1)$ & $(0,1)$

Point of inflection: only $\boxed{x=0}$ $(0,f(0))$

Radical

domain: $(-\infty, \infty)$

$f(0) = 0 + 0^{1/3} = 0 + 0 = 0$

Example: Find the points of inflection and determine the concavity of $\boxed{f(x) = x + x^{1/3}}$.

$f'(x) = 1 + \frac{1}{3} \cdot x^{-2/3}$

$f''(x) = \frac{1}{3} \cdot \frac{-2}{3} \cdot x^{-5/3} = \frac{-2}{9 \cdot x^{5/3}}$

$f'' = 0$? Never

f'': undefined? $\boxed{x = 0}$

f'': $+$ undef $-$

f: $\cup$ $\boxed{0}$ $\cap$

f is concave up on: $(-\infty, 0)$

$\boxed{\text{PoI at } x = 0}$

Example: Given that the function $\boxed{f(x) = x^4 + Ax^2 + Bx}$ has a ~~relative minimum~~ local extreme at $x = -1$ and a point of inflection at $x = 2$, find the values of A and B.

$\underbrace{\text{local extreme at}}$ $x = -1$.

$x = -1$ is a critical point

$f'(-1) = 0$

$f'(x) = 4x^3 + 2Ax + B$

$f'(-1) = 4(-1) + 2A(-1) + B = 0$

$-4 - 2A + B = 0$

$x = 2$ is a PoI means $f''(2) = 0$

$f''(x) = 12x^2 + 2A$

$f''(2) = 12 \cdot (2)^2 + 2A = 0 \Rightarrow 2A = -48$

$A = -24$

$$-4 - 2A + B = 0$$

$$-4 - 2(-24) + B = 0$$

$$\Rightarrow \quad -4 + 48 + B = 0$$

$$44 + B = 0$$

$$\Rightarrow \quad \boxed{B = -44}$$

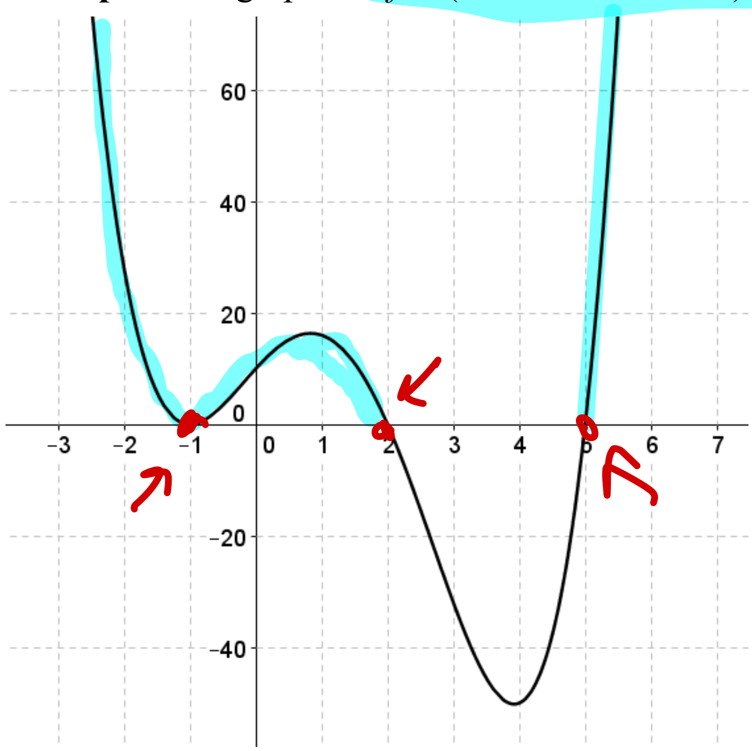
Exercise: Find the points of inflection for the function $f(x) = 3\cos(2x)$ over the interval $x \in [0, \pi]$.

Exercise: Find the points of inflection and determine the concavity of $f(x) = x - \cos(x),\ x \in [0, 2\pi]$.

Graphs of f, f' and f'':

When the graph of the second derivative is given, we can gather information about whether f' is increasing or decreasing, and whether f is concave up or down. We can also figure out the points of inflection for f.

Example: The graph of f'' (second derivative!) is given.



PoI of $f(x)$

~~1~~, $\boxed{2}$, $\boxed{5}$

$\boxed{x=2, \ x=5}$

When is $\boxed{f(x) \text{ concave up?}}$ ← if $f''>0$ $(-\infty, 2)$ & $(5, \infty)$

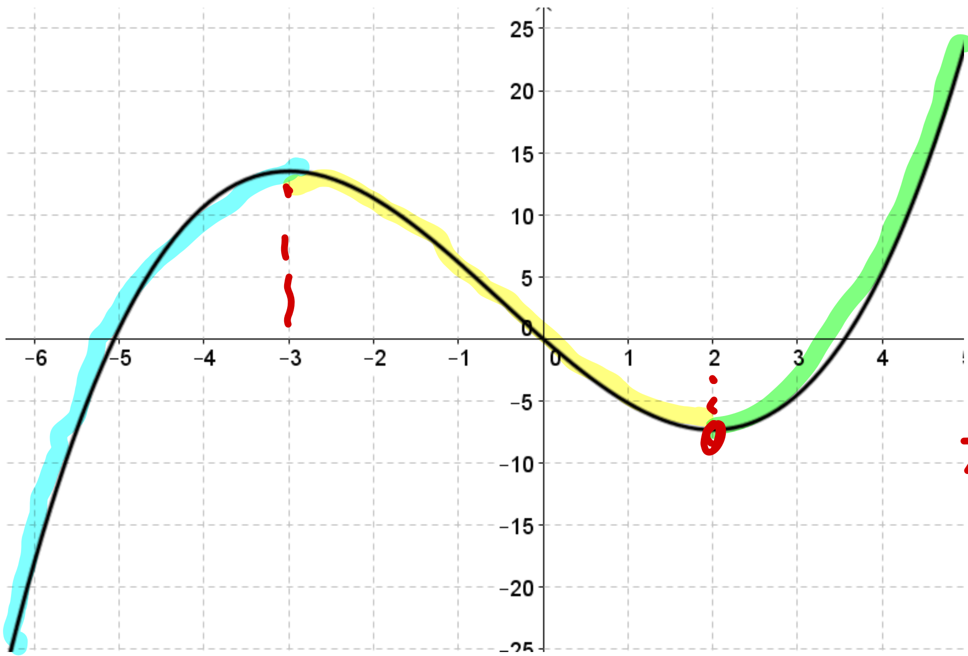
When is $f(x)$ concave down? ← if $f''<0$: $(2,5)$

What are the points of inflection of $f(x)$?

$\boxed{\text{When is } f'(x) \text{ increasing?}}$

f' is increasing if $(f')' = f'' > 0$

Example: The graph of $f'(x)$ (first derivative!) is given.



-3 $\quad$ 2

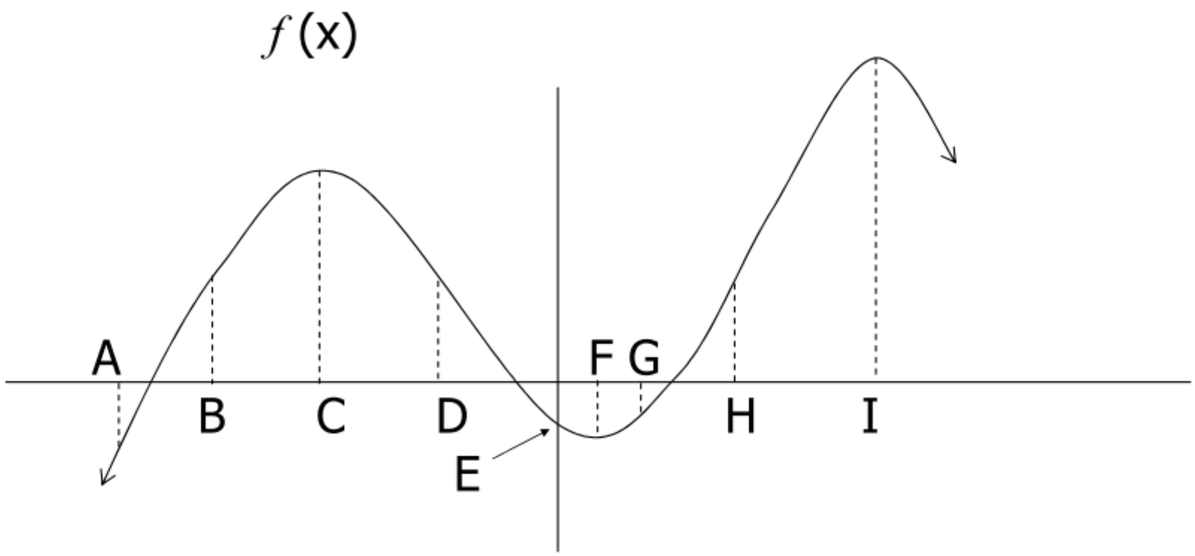
When is $\boxed{f(x) \text{ concave up?}}$ if $\boxed{f' \nearrow}$ $\Leftarrow$ $(-\infty,-3)$ & $(2,\infty)$

When is $f(x)$ concave down? $\quad (-3, 2)$

What are the x-coordinates of points of inflection of $f(x)$?

PoI of f $\quad \boxed{x=-3,\ x=2}$

EXERCISE:



Using the graph of f(x), complete the chart by indicating whether the functions are positive, negative, or zero at the indicated points.

D and H are POI's.

	f(x)	f'(x)	f''(x)
A			
B			
C			
D			
E			
F			
G			
H			
I			

3.5 is over