Completed

Math 2413- Calculus I

Dr. Melahat Almus

Email: malmus@uh.edu

- Check CASA calendar for due dates.
- Bring "blank notes" to class. Completed notes will be posted after class.
- Do your best to attend every lecture and lab.
- Study after every lecture; work on the quiz covering the topic we cover on the lecture immediately afterwards. Retake your quizzes for more practice.
- Get help when you need help; bring your questions to the labs, or my office hours. We also have tutoring options on campus.
- Respect your friends in class; stay away from distractive behavior. Do your best to concentrate on the lecture.
- If you email me, mention the course code in the subject line. Email is the best way to communicate with me outside of class. Teams chat messages are not monitored or replied to.

$\cancel{(-2)^x}$ $\quad$ $\cancel{1^x}$ $\quad$ $\left(\frac{1}{2}\right)^x$

$f(x) = 2^x$ $\quad$ $g(x) = e^x$

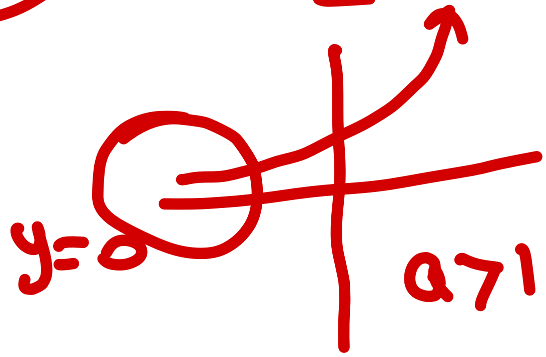
Section 4.2 - The Exponential Function

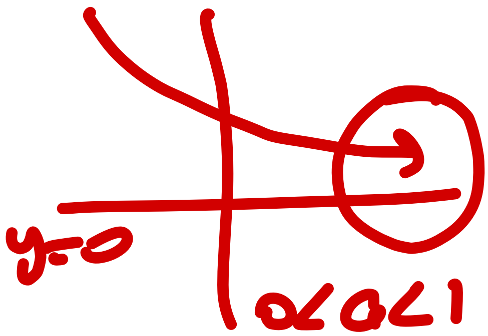
Definition: A function in the form $f(x) = a^x$ (where $a > 0, a \neq 1$) is called an **exponential function**.

Domain: All real numbers;

Range: $(0,\infty)$

$\underline{\underline{a^x}}$



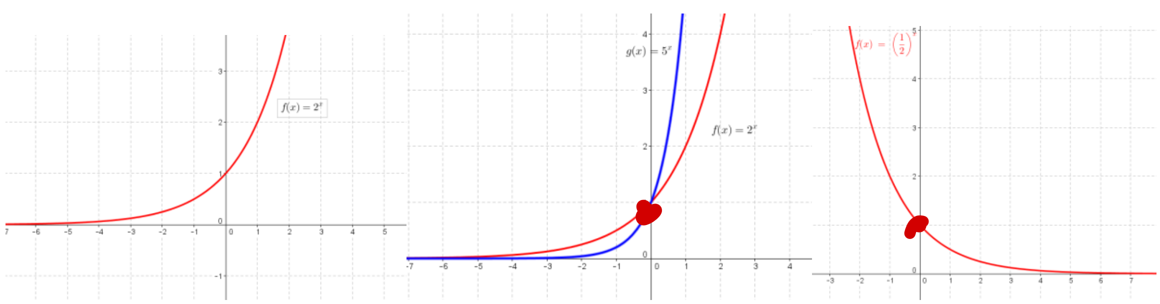


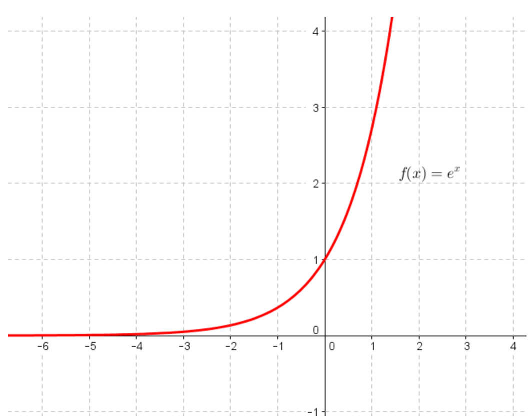
$f(x) = 2^x$

$f(x) = 5^x$

$f(x) = \left(\frac{1}{2}\right)^x$

$f(x) = e^x$





$2^x + 5$

$e^x - 1$

e^{2x} e^{x^2+1}

Derivatives of Exponential Functions:

The exponential function with base "e" has the unique property that it is its own derivative.

$$\frac{d}{dx}\left(e^x\right) = e^x$$

If the exponent is u (a function in terms of x), then we use the chain rule:

$$\frac{d}{dx}\left(e^u\right) = e^u \cdot \frac{du}{dx}$$

$=$ $(e^u)' = e^u \cdot u'$

Example: Differentiate the following functions.

(a) $y = 5e^x$ $\Rightarrow$ $y' = 5 \cdot e^x$

deriv. of exponent

(b) $y = e^{2x}$ $\Rightarrow$ $y' = e^{2x} \cdot (2) = \boxed{2e^{2x}}$

(c) $f(x) = 4e^{x^2}$ $\Rightarrow$ $y' = 4 \cdot e^{x^2} \cdot (2x) = \boxed{8x \cdot e^{x^2}}$

$=$

(d) $g(x) = 2e^{\sqrt{x}}$

$\Rightarrow$ $g'(x) = 2 \cdot e^{\sqrt{x}} \cdot \left(\frac{1}{2\sqrt{x}}\right) = \frac{e^{\sqrt{x}}}{\sqrt{x}}$

$g'(1) = \frac{e^1}{1} = e$

$g'(4) = \frac{e^2}{2}$

Note: $e^0 = 1$

Example: Given, $g(x) = e^{\sin(x)} + e^{-2x}$, find $g'(0) = ?$

deriv of exp.

$$g'(x) = e^{\sin(x)} \cdot (\cos(x)) \boxed{+} e^{-2x} \cdot (-2)$$

$$g'(0) = e^0 \cdot \cos(0) + e^0 \cdot (-2)$$

$$g'(0) = 1 - 2 = \boxed{-1}$$

EHW2

Example: For $f(x) = xe^{-x}$, Find the slope of the tangent line at $x = 2$.

$f'(2) = ?$

$$f(x) = x \cdot e^{-x}$$

product rule!

$$f'(x) = 1 \cdot e^{-x} + x \cdot e^{-x} \cdot (-1)$$

$$f'(2) = e^{-2} - 2 \cdot e^{-2} = -e^{-2} = \frac{-1}{e^2}$$

$$A - 2A = -A$$

$f(0) = 2 + e^0 = 2 + 1 = 3$

product rule

Point $(0,3)$

Example: For $f(x) = 2 + e^{4x\cos(2x)}$, find the equation of the tangent line at x=0:

$m = f'(0)$

$f'(x) = e^{4x\cdot\cos(2x)} \cdot [4\cos(2x) - 8x\sin(2x)]$

$f'(0) = e^0 \cdot [4\cos(0) - 8\cdot 0\cdot \sin(0)]$

$f'(0) = 1\cdot 4 = 4$

$m_{tangent} = 4$ $(0,3)$

$y - 3 = 4(x - 0) \Rightarrow y - 3 = 4x$

$\boxed{y = 4x + 3}$

Example: $f(x) = \dfrac{e^x + e^{-x}}{1 + e^{2x}}$; $f'(x) = ?$

$$f'(x) = \frac{(e^x - e^{-x})(1 + e^{2x}) - (e^x + e^{-x})\cdot(2e^{2x})}{(1 + e^{2x})^2}$$

useful: $(e^{mx})' = e^{mx} \cdot m$

$(e^{2x})' = 2e^{2x}$

$(e^{5x})' = 5e^{5x}$ $(e^{-x})' = -e^{-x}$

divide

$$\frac{e^m}{e^n} = e^{m-n}$$

Exercise: $f(x) = \frac{\boxed{e^{2x}} + e^{-2x} + 1}{\boxed{e^x}}$; $f'(x) = ?$

easier if: $f(x) = e^x + e^{-3x} + e^{-x}$

$\downarrow$

$f'(x) =$

Different BASES:

Fact: The derivative of an exponential function is a constant time the function itself.

$$\frac{d}{dx}\left(a^x\right) = a^x \ln a$$

$(8^x)' = 8^x \cdot \ln(8)$

If the exponent is u (a function in terms of x), then we use the chain rule:

$$\frac{d}{dx}\left(a^u\right) = a^u \ln a \cdot \frac{du}{dx}$$

$(a^u)' = a^u \cdot \ln(a) \cdot u'$

Example: Differentiate the following functions.

(a) $y = 5^x$ $\Rightarrow$ $y' = 5^x \cdot \ln(5)$

(b) $y = 6^{2x+1}$

$\downarrow$

$y' = 6^{2x+1} \cdot \ln(6) \cdot (2)$

(c) $f(x) = 4^{x^3+x}$

der. of exp.

$$f'(x) = 4^{x^3+x} \cdot \ln(4) \cdot (3x^2+1)$$

$$f'(0) = 4^0 \cdot \ln(4) \cdot (1) = \ln(4)$$

$$f'(1) = 4^2 \cdot \ln(4) \cdot (4) = 64 \cdot \ln(4)$$

(d) $g(x) = e^{-x} + 10^{\sqrt{x}}$

$$g'(x) = -e^{-x} + 10^{\sqrt{x}} \cdot \ln(10) \cdot \left(\frac{1}{2\sqrt{x}}\right)$$

Example: Given: $g(x) = x \cdot 5^{\sin(2x)}$, find the slope of the tangent line at $x = 0$.

Product rule!

$$g'(x) = 1 \cdot 5^{\sin(2x)} + x \cdot 5^{\sin(2x)} \cdot \ln(5) \cdot 2\cos(2x)$$

$$g'(0) = 5^0 + 0 \cdot \underbrace{5^0 \cdot \ln(5) \cdot 2 \cdot \cos(0)}$$

$$g'(0) = 1 + 0 = 1$$

Exercise: Find the local extreme points of $g(x) = x \cdot e^{-2x}$. Find the intervals over which the function is increasing.

Section 4.2 Formulas:

$$\frac{d}{dx}\left(e^{x}\right) = e^{x} \qquad \frac{d}{dx}\left(e^{u}\right) = e^{u} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}\left(a^{x}\right) = a^{x} \ln a \qquad \frac{d}{dx}\left(a^{u}\right) = a^{u} \ln a \cdot \frac{du}{dx}$$

Q# **If** $f(x) = e^{x^2+x}$**, find** $f'(1) = ?$

a. 3
b. 3e
c. e^2
d. $3e^2$
e. None

Question# **If** $f(x) = 5^{(x^2+3x)}$**, find** $f'(0) = ?$

a. 1
b. ln(5)
c. 5 ln(5)
d. 3ln(5)

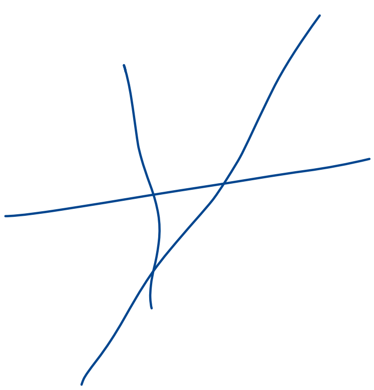
e. None

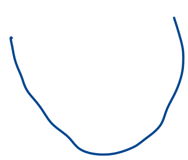
Q# **If** $f(x) = 4^{\sin x}$**, find** $f'(0) = ?$

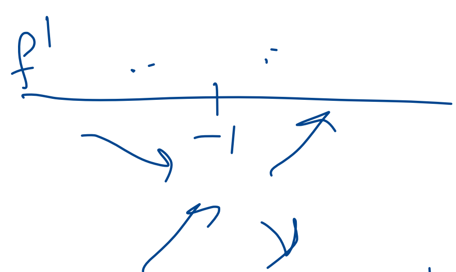
a. 1
b. ln(4)
c. 4 ln(4)
d. 4
e. None

$f(x) = ???$

has local max at $x = 5$



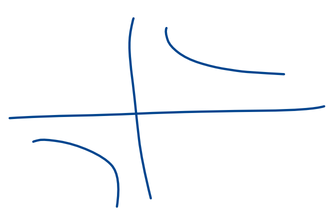




$\sqrt[3]{x-5}$

$-|x-5|$

$\frac{1}{x-5}$



$f''(x) = 3x^2 - 2x - 10$

$f''(-1) = 3 + 2 - 10 = -5 < 0$