Math 2413- Calculus I

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- Check CASA calendar for due dates.
- Bring "blank notes" to class. Completed notes will be posted after class.
- Do your best to attend every lecture and lab.
- Study after every lecture; work on the quiz covering the topic we cover on the lecture immediately afterwards. Retake your quizzes for more practice.
- Get help when you need help; bring your questions to the labs, or my office hours. We also have tutoring options on campus.
- Respect your friends in class; stay away from distractive behavior. Do your best to concentrate on the lecture.
- If you email me, mention the course code in the subject line. Email is the best way to communicate with me outside of class. Teams chat messages are not monitored or replied to.

Section 4.4 - Inverse Trigonometric Functions

In section 4.1, we learned that in order to have an inverse, a function must be one-to-one. Since trigonometric functions are periodic, they are NOT one-to-one. To define the inverse trig functions, we must restrict the usual domains.

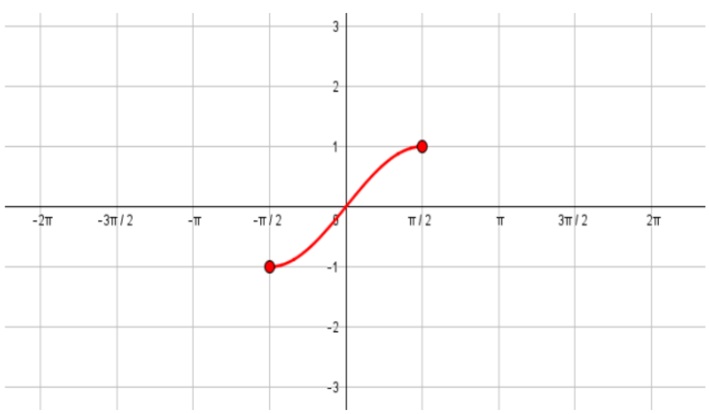
Here is the graph of $f(x) = \sin(x)$:

Domain: $(-\infty, \infty)$

Range: $[-1,1]$

If the function is not one-to-one, we run into problems when we consider the inverse of the function. What we want to do with the sine function is to restrict the values for sine. When we make a careful restriction, we can get something that IS one-to-one.

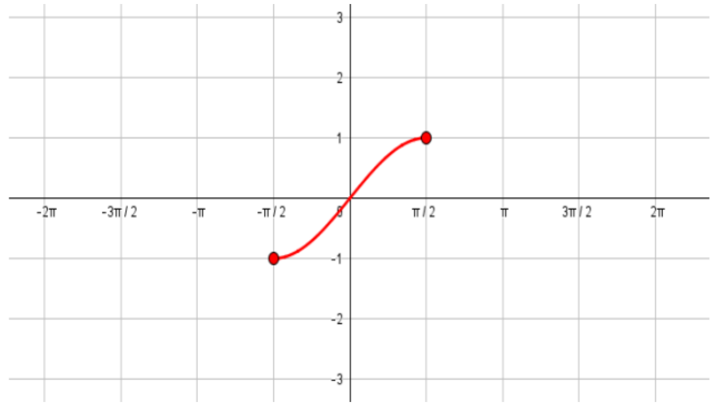
If we limit the function to the interval $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$, the graph will look like this:



Restricted Sine function

Domain: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Range: $[-1,1]$



On this limited interval, we have a one-to-one function; we might consider finding its inverse.

Here is the graph of Inverse Sine function:

$f(x) = \sin^{-1}(x) = \arcsin(x)$

Domain: $[-1,1]$

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

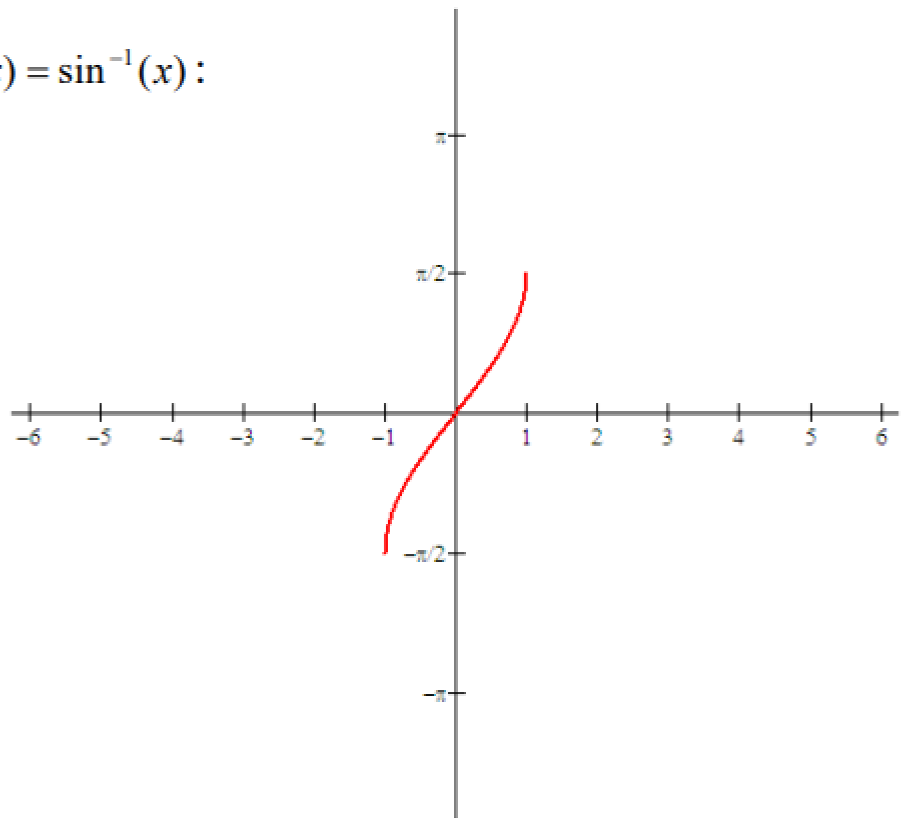
Key points on the graph:

$(0,0)$

$\left(1, \frac{\pi}{2}\right)$

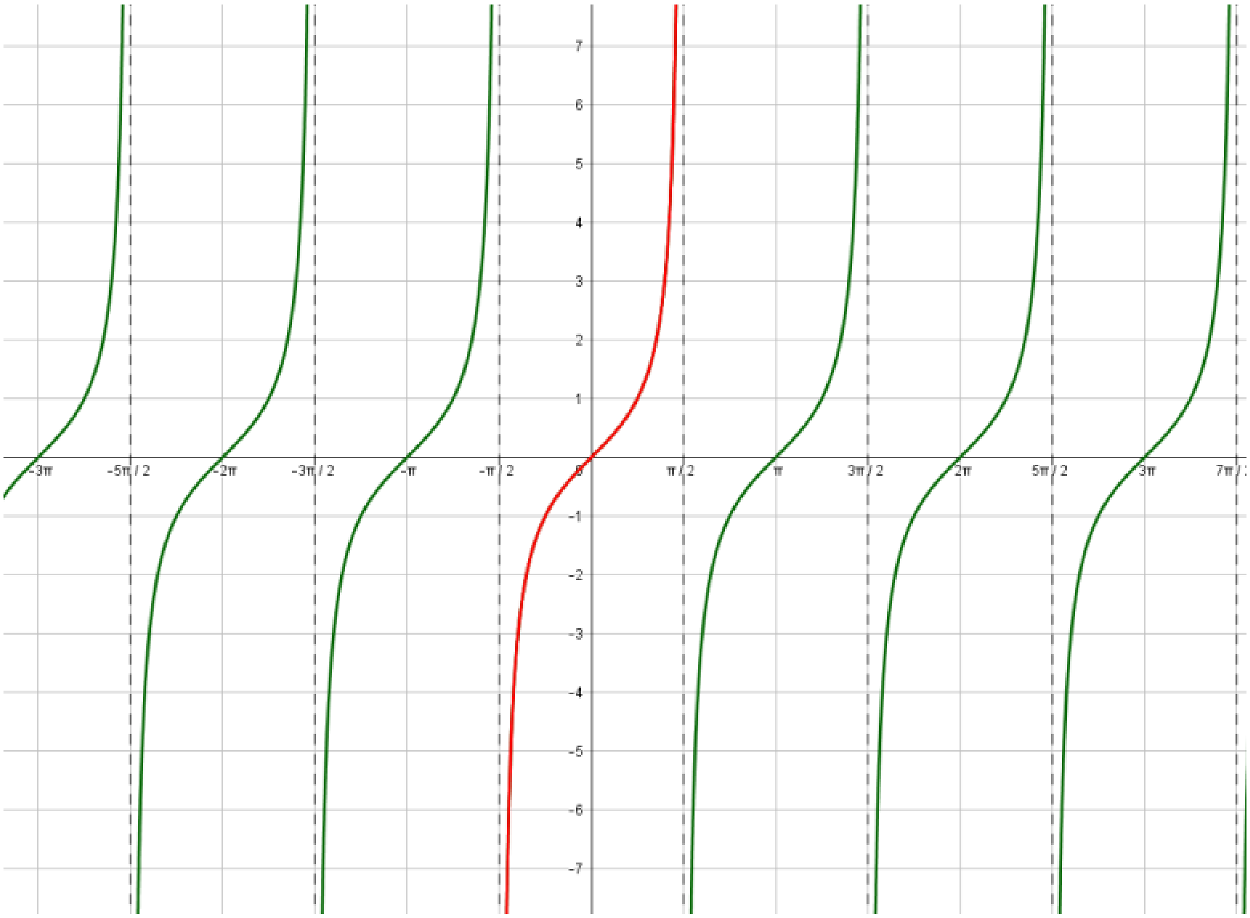
$\left(-1, -\frac{\pi}{2}\right)$

$f(x) = \sin^{-1}(x)$:



INVERSE TANGENT FUNCTION

Here's the graph of $f(x) = \tan(x)$. Is it one-to-one?



If we restrict the function to the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, then the restricted function IS one-to-one.

Note that $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ are excluded; they are vertical asymptotes for the tangent function.

Restricted Tangent function

Domain: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

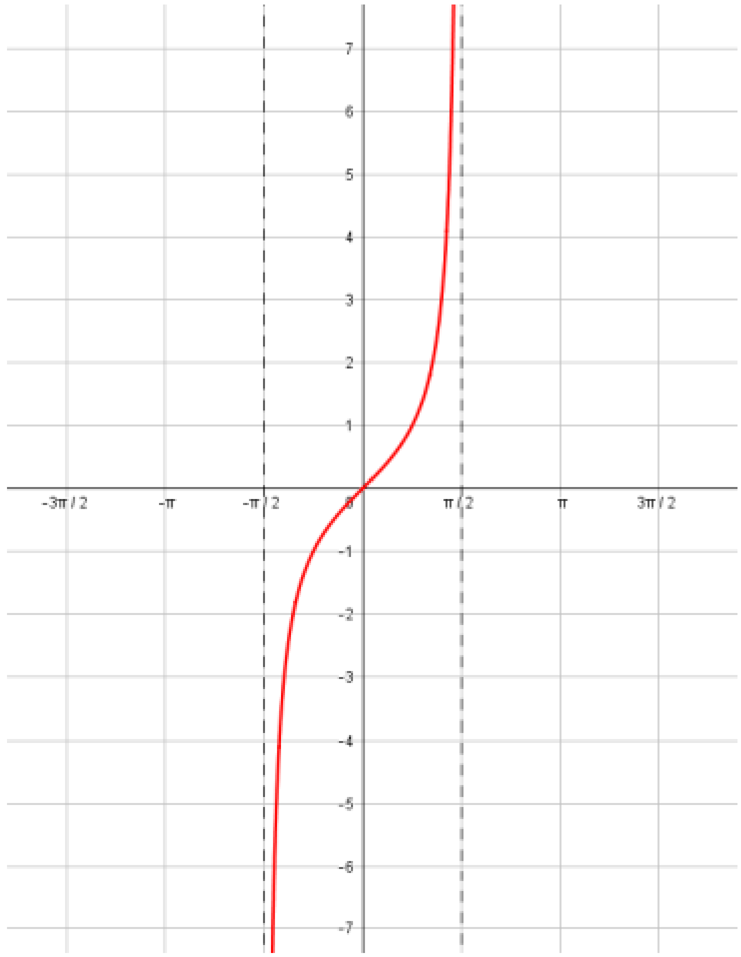
Range: $(-\infty, \infty)$

Key points on the graph:

$(0,0)$

$\left(\frac{\pi}{4}, 1\right)$

$\left(-\frac{\pi}{4}, -1\right)$



Restricted Tangent Function	**Inverse Tangent Function**
$f(x) = \tan(x)$	$f^{-1}(x) = \tan^{-1}(x) = \arctan(x)$
Domain: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$	Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Graph of inverse tangent function:

$y = \arctan(x)$

Domain: $(-\infty, \infty)$

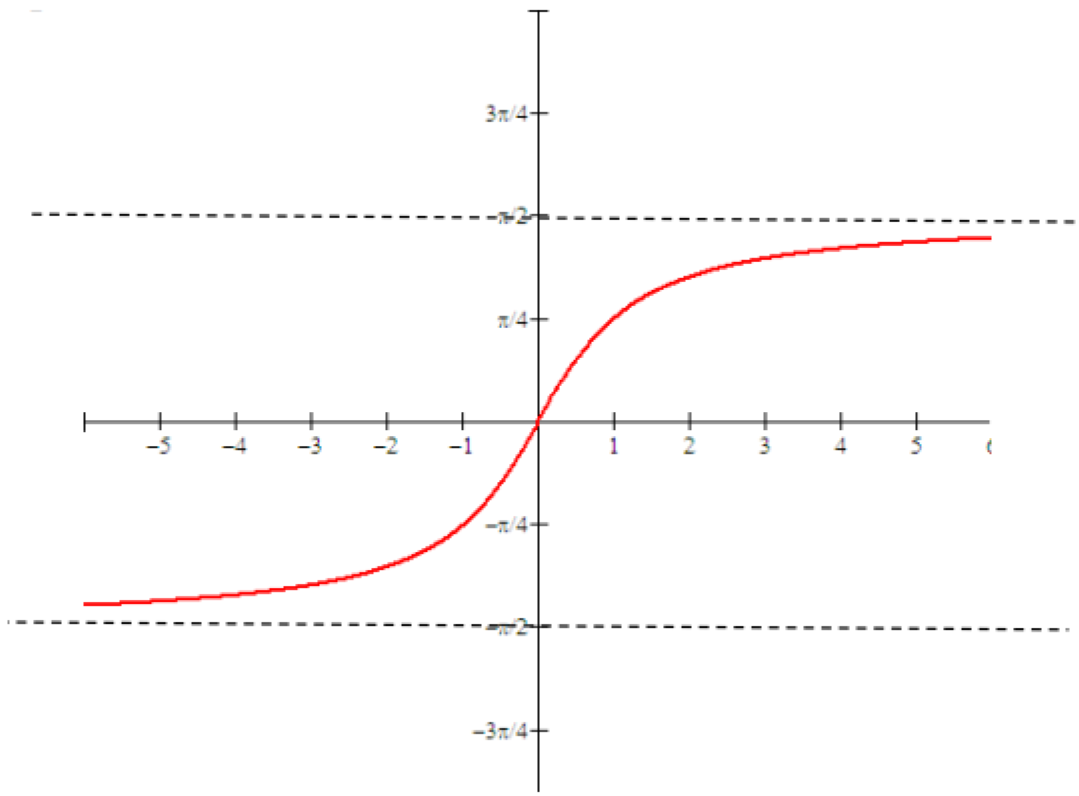
Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Key points on the graph:

$(0,0)$

$\left(1, \frac{\pi}{4}\right)$

$\left(-1, -\frac{\pi}{4}\right)$

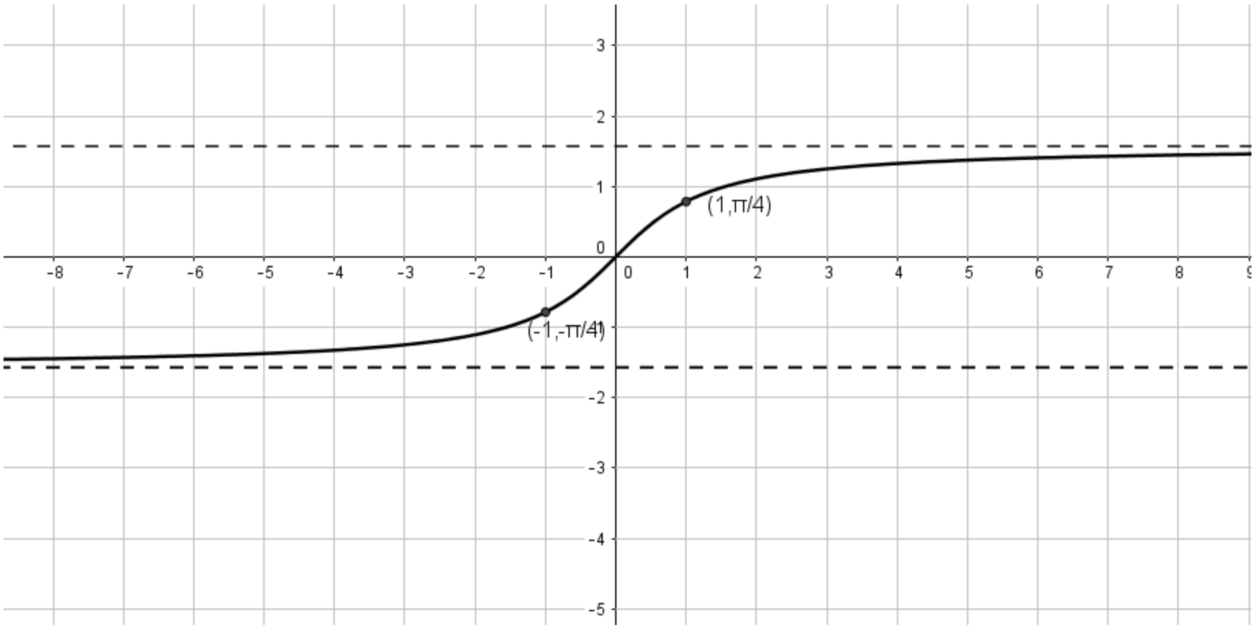


Observe from the graph that negative inputs produce negative outputs; positive inputs produce positive outputs.

Important: This graph has two horizontal asymptotes:

$y = -\frac{\pi}{2}$ and $y = \frac{\pi}{2}$ are horizontal asymptotes for the arctangent function.

$$\arctan(x):(-\infty,\infty)\to\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$$



INVERSE SECANT FUNCTION

SECANT FUNCTION: $g(x) = \sec(x)$

Period: 2π

Vertical Asymptote:$2k\pi$

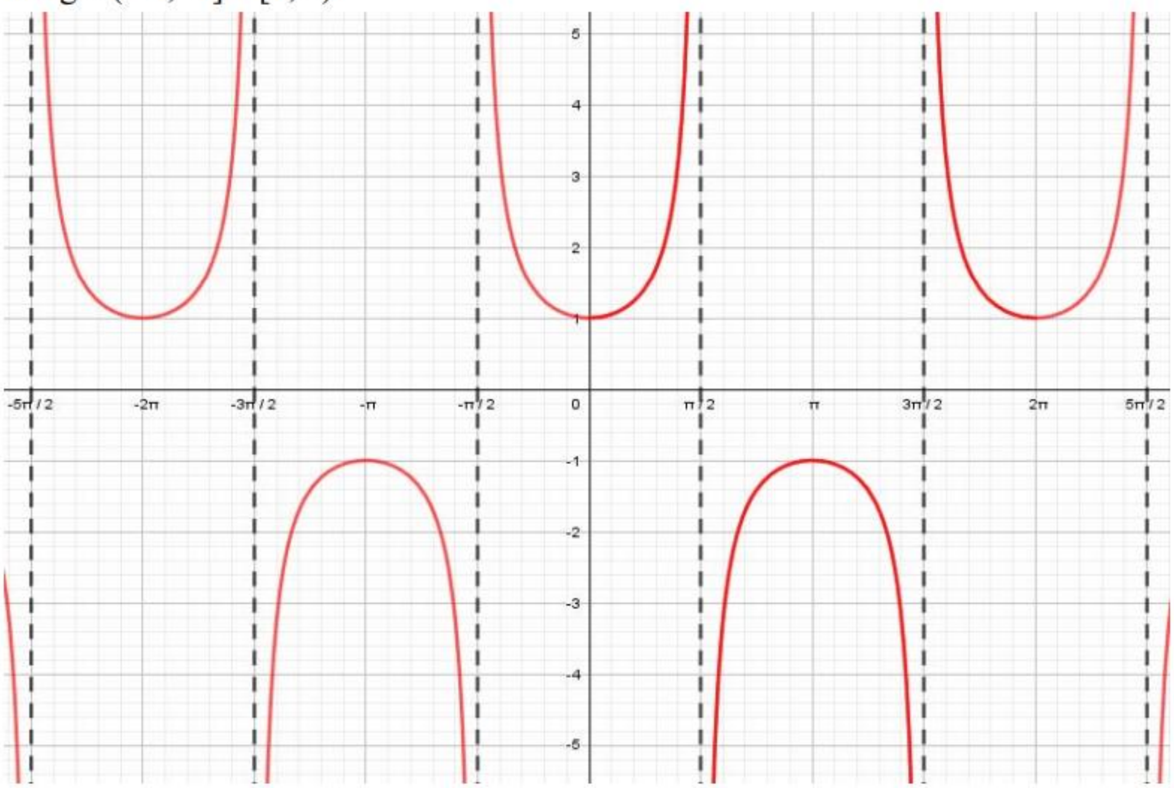
$x = k\pi / 2$ is an odd integer

x-intercepts: None

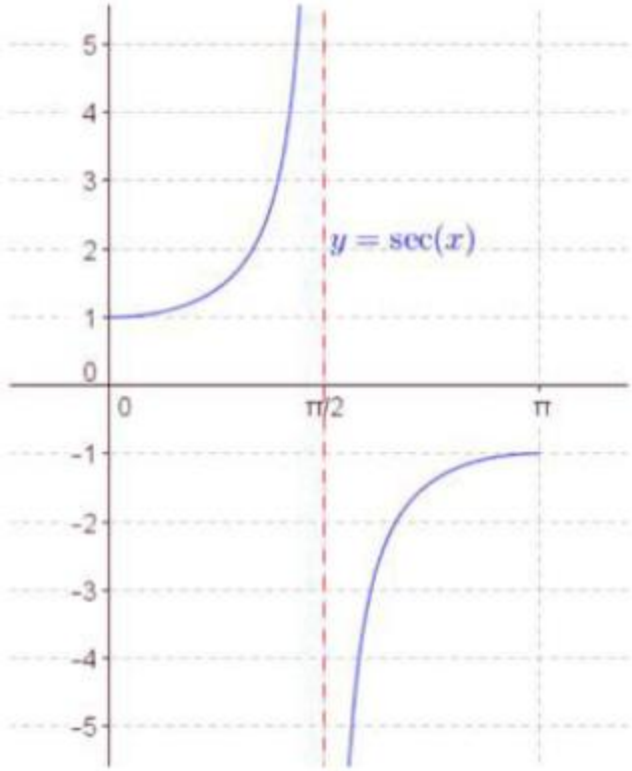
y-intercept: (0, 1)

Domain: $x \neq k\pi / 2$, k is an odd integer

Range: $(-\infty, -1] \cup [1, \infty)$



Restricted Secant function:



Domain: $\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right]$

Range: $(-\infty,-1]\cup[1,\infty)$

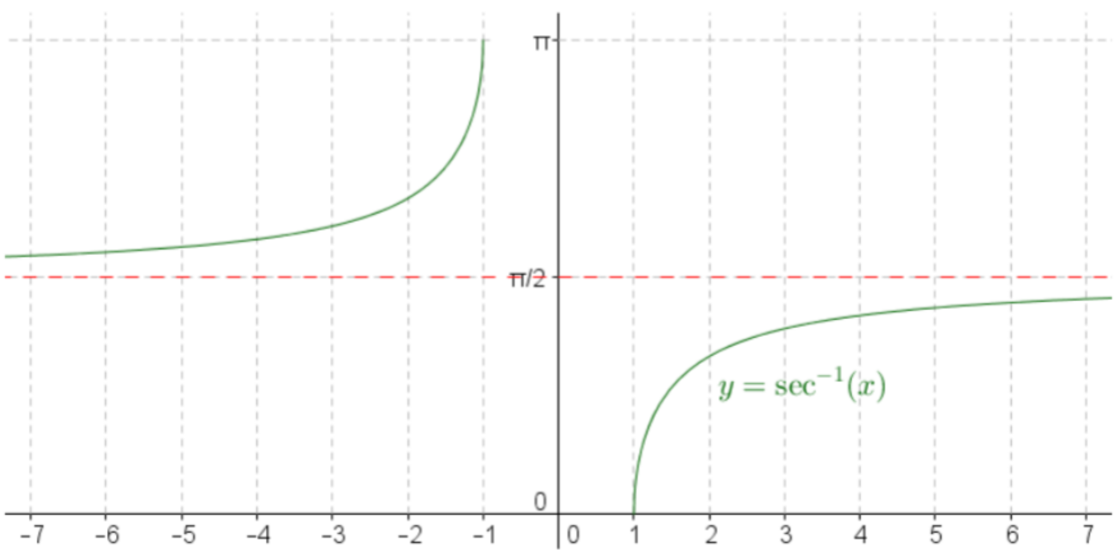
Vertical asymptote: $x=\frac{\pi}{2}$

Inverse Secant Function:

$$\operatorname{arcsec}(x):(-\infty,-1]\cup[1,\infty)\to\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right].$$

Horizontal asymptote: $y=\frac{\pi}{2}$

The inverse function $\sec^{-1}(x)$ has the following graph.



EXERCISES From PreCalculus – solve before class!!!

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$$

$$\sin^{-1}\left(\frac{1}{2}\right) =$$

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$$

$$\arcsin\left(\frac{-\sqrt{3}}{2}\right) =$$

$$\arctan(1) =$$

$$\arctan(-1) =$$

REMARK: Note that domain of sine and inverse cosine functions are $[-1,1]$. That is, numbers that are not in this interval can NOT be inputs. The following are UNDEFINED:

$\arcsin(5)$: undefined

$\arccos(4)$: undefined

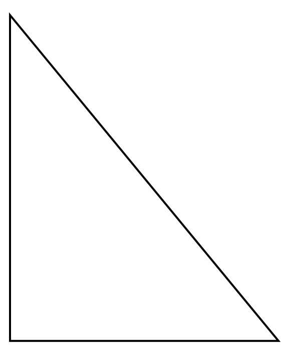
$\arcsin(-2)$: undefined.

For arctangent, the input can be any real number: $\arctan(5)$ is defined.

Example:

$y = \arcsin\left(x\right)$; express $\cos(y)$ in terms of "x".

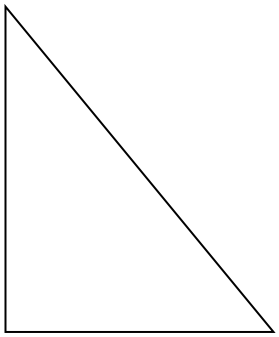
Example: If $y = \arcsin(x)$, Find y'.



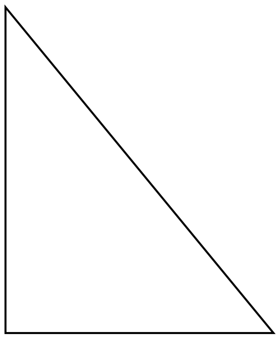
Fact: If $f(x) = y = \arcsin(x)$ then $y' =$

How about:

- $f(x) = y = \arctan(x)$ find $y' =$



- $f(x) = y = \operatorname{arcsec}(x)$ find $y' =$



Formulas (u is a function of x):

$$\frac{d}{dx}\left[\arcsin x\right] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left[\arcsin u\right] = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}\left[\arctan x\right] = \frac{1}{1+x^2} \qquad \frac{d}{dx}\left[\arctan u\right] = \frac{u'}{1+u^2}$$

$$\frac{d}{dx}\left[\operatorname{arcsec} x\right] = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\left[\operatorname{arcsec} u\right] = \frac{u'}{|u|\sqrt{u^2-1}}$$

Example: Differentiate: $y = 2\arcsin\left(x\right)$.

Example: Differentiate: $y = \arcsin\left(6x+1\right)$.

Example: Differentiate: $y = 5\arctan(x)$

Example: Differentiate: $y = \tan^{-1}\left(2\sqrt{x}\right)$

Example: Differentiate: $f\left(x\right) = e^{\arctan(5x)}$

Example: Differentiate: $y = \sec^{-1}\left(x^2 + 2\right)$

Example: Differentiate: $y = \arcsin\left(\ln x\right) + \arctan\left(e^x\right)$

Example: Give the domain of $g(x) = \arcsin\left(\frac{e^x}{2}\right)$ and find the equation for the tangent line to the graph of this function at x = 0.

Example: Find the slope of the tangent line at $x=0$: $f(x)=e^{2x}\arctan(4x)$

Exercise: Differentiate: $y=\ln(\arcsin(x))+\arctan(4x)$

Popper#

Question# Give the derivative of $\mathrm{f}(x)=\arctan(3x)$.

a. $\mathrm{f}'(x)=\dfrac{3}{1+3x^2}$

b. $\mathrm{f}'(x)=\dfrac{3}{1+9x^2}$

c. $\mathrm{f}'(x)=\dfrac{1}{1+9x^2}$

d. $\mathrm{f}'(x)=3\,\mathrm{arc}\sec^2(3x)$

e. None

Question# Give the derivative of $\mathrm{f}(x)=\arcsin(5x)$.

a. $\mathrm{f}'(x)=\dfrac{5}{1+25x^2}$

b. $f'(\mathrm{x})=\dfrac{5}{\sqrt{1-5\mathrm{x}^2}}$

c. $\mathrm{f}'(x)=\dfrac{5}{\sqrt{1-25x^2}}$

d. $\mathrm{f}'(x)=5\arccos 5x$

e. None

The following table lists the most common "inverse trig" function values. Make sure you know these and understand why these are true. If you know the unit circle, you know the values listed here; you just need to think "backwards" and know the range restrictions.

$\arcsin(-1) = -\frac{\pi}{2}$	$\arccos(-1) = \pi$
$\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$	$\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$
$\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$	$\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$
$\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$	$\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$
$\arcsin(0) = 0$	$\arccos(0) = 0$
$\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$	$\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$
$\arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$	$\arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$
$\arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$	$\arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$
$\arcsin(1) = \frac{\pi}{2}$	$\arccos(1) = 0$

$\arctan(-1) = -\frac{\pi}{4}$
$\arctan(0) = 0$
$\arctan(1) = \frac{\pi}{4}$
$\arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$
$\arctan\left(\sqrt{3}\right) = \frac{\pi}{3}$
$\arctan\left(-\sqrt{3}\right) = -\frac{\pi}{3}$
$\arctan\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$