Math 2413- Calculus I

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- Check CASA calendar for due dates.
- Bring “blank notes” to class. Completed notes will be posted after class.
- Do your best to attend every lecture and lab.
- Study after every lecture; work on the quiz covering the topic we cover on the lecture immediately afterwards. Retake your quizzes for more practice.
- Get help when you need help; bring your questions to the labs, or my office hours. We also have tutoring options on campus.
- Respect your friends in class; stay away from distractive behavior. Do your best to concentrate on the lecture.
- If you email me, mention the course code in the subject line. Email is the best way to communicate with me outside of class. Teams chat messages are not monitored or replied to.

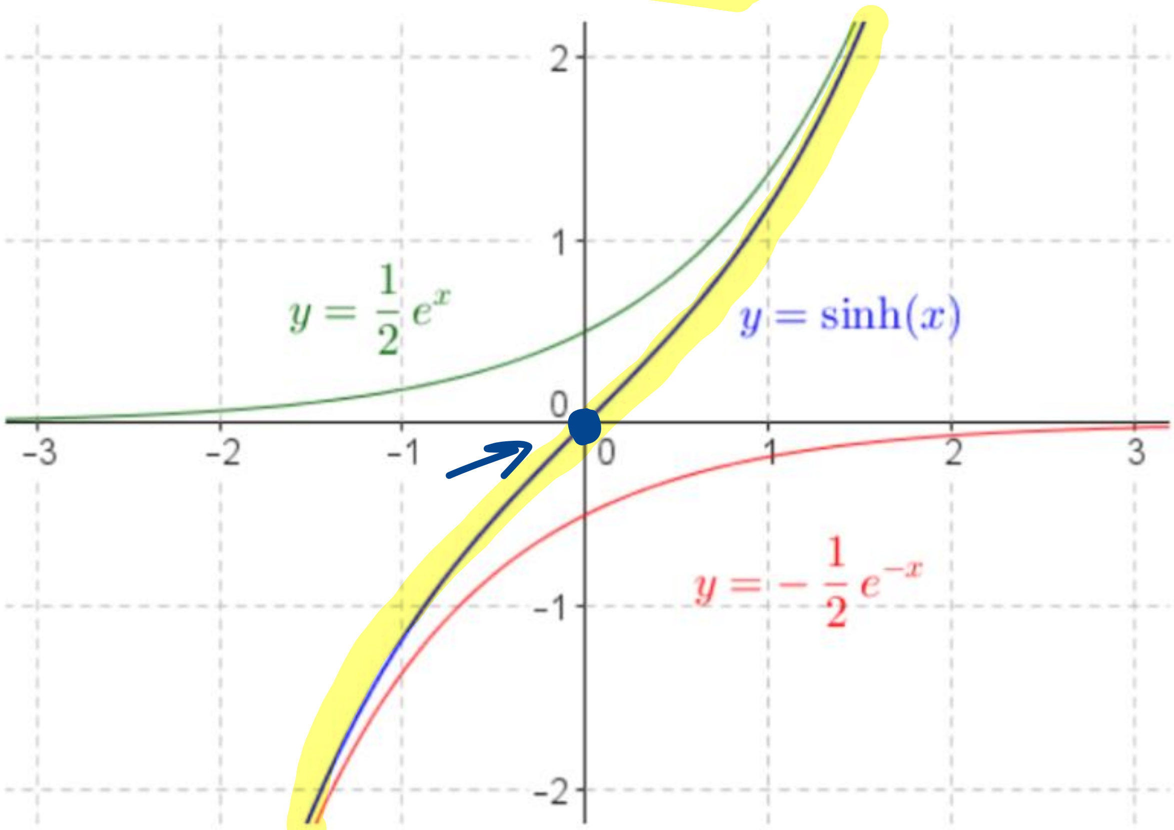
$f(x) = e^x$

$g(x) = e^{-x}$

Section 4.5 – Hyperbolic Functions

Primary Definitions

Hyperbolic sine: $\sinh(x) = \dfrac{e^x - e^{-x}}{2}$



Observe: $\sinh(0) = 0$; hyperbolic sine is increasing. It is an ODD function.

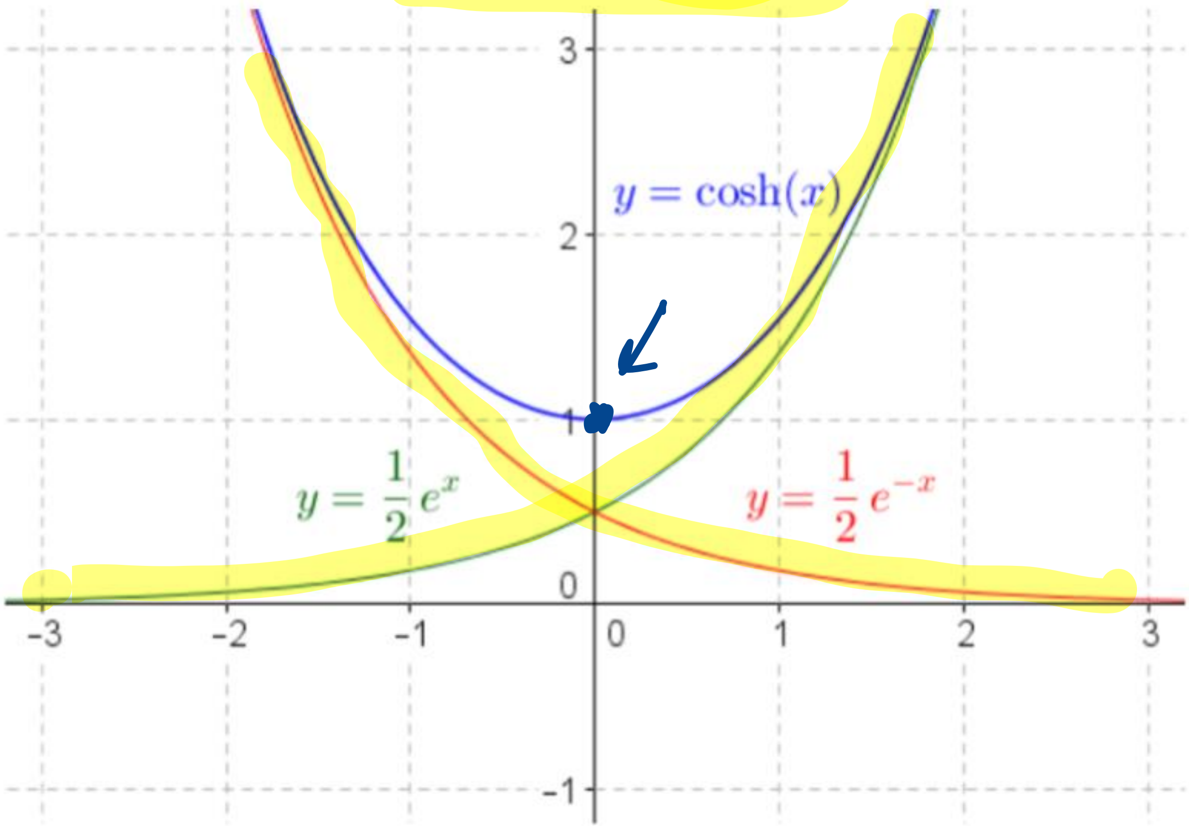
To evaluate this function at different values, use the definition. For example:

$f(1) = \sinh(1) = \dfrac{e - e^{-1}}{2} = \dfrac{1}{2}e - \dfrac{1}{2e}$

$\sinh(0) = \dfrac{e^0 - e^{-0}}{2} = 0$

$\cosh(0) = \frac{e^0 + e^{-0}}{2} = \frac{1+1}{2} = 1$

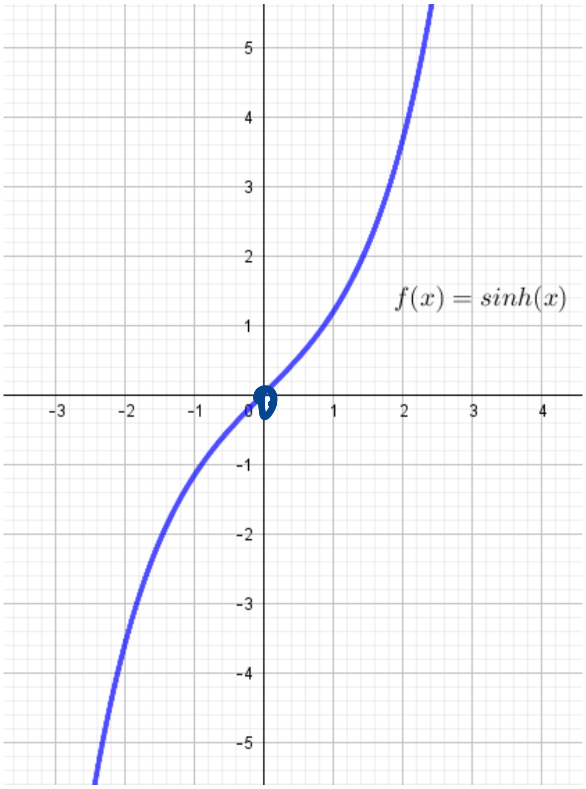
Hyperbolic cosine: $\boxed{\cosh(x) = \frac{e^x + e^{-x}}{2}}$



Observe: $\cosh(0) = 1$. Hyperbolic cosine is decreasing on $(-\infty, 0)$, increasing on $(0, \infty)$. Its concave up on its domain. It is an **even** function.

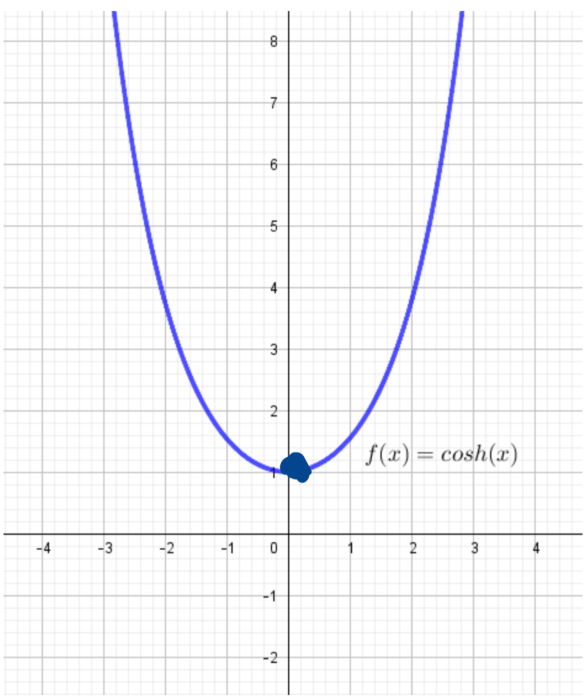
To evaluate this function at different values, use the definition. For example:

$f(2) = \cosh(2) = \frac{e^2 + e^{-2}}{2}$



odd

(0,0)



even

(0,1)

$$\tanh(x) = \frac{\frac{e^x - e^{-x}}{2}}{\frac{e^x + e^{-x}}{2}} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

The four remaining hyperbolic functions are defined as you would expect given their names. That is:

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} \qquad \coth(x) = \frac{\cosh(x)}{\sinh(x)}$$

$$\operatorname{sech}(x) = \frac{1}{\cosh(x)} \qquad \operatorname{csch}(x) = \frac{1}{\sinh(x)}$$

$$= \frac{2}{e^x + e^{-x}}$$

If you need to evaluate these functions, use the definitions.

Examples:

$$\tanh(0) = \frac{\sinh(0)}{\cosh(0)} = \frac{0}{1} = 0$$

$$\tanh(1) = \frac{\sinh(1)}{\cosh(1)} = \frac{\frac{e - e^{-1}}{2}}{\frac{e + e^{-1}}{2}} = \frac{e - e^{-1}}{e + e^{-1}}$$

$\sec h(0) = \frac{1}{\cosh(0)} = \frac{1}{1} = 1$, and so on.

$(e^x)' = e^x \qquad (e^{-x})' = -e^{-x}$

DERIVATIVES OF HYPERBOLIC FUNCTIONS

What is the derivative of hyperbolic sine function?

$$f(x) = \sinh(x) = \frac{e^x - e^{-x}}{2} \rightarrow f'(x) = \frac{e^x + e^{-x}}{2} = \cosh(x)$$

Reason:

$$\frac{e^x - e^{-x}}{2} = \frac{1}{2}e^x - \frac{1}{2}e^{-x} \rightarrow \frac{d}{dx}\left(\frac{1}{2}e^x - \frac{1}{2}e^{-x}\right) = \frac{1}{2}e^x + \frac{1}{2}e^{-x} = \frac{e^x - e^{-x}}{2}$$

What is the derivative of hyperbolic cosine function?

$$f(x) = \cosh(x) = \frac{e^x + e^{-x}}{2} \rightarrow f'(x) = \frac{e^x - e^{-x}}{2} = \sinh(x)$$

Derivatives of 6 basic hyperbolic functions:

$$\frac{d}{dx}\left[\sinh(x)\right]=\cosh(x)$$
$$\frac{d}{dx}\left[\cosh(x)\right]=\sinh(x)$$
$$\frac{d}{dx}\left[\tanh(x)\right]=\operatorname{sech}^2(x)$$

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$$\frac{d}{dx}\left[\operatorname{sech}(x)\right]=-\tanh(x)\operatorname{sech}(x)$$
$$\frac{d}{dx}\left[\coth(x)\right]=-\operatorname{csch}^2(x)$$
$$\frac{d}{dx}\left[\operatorname{csch}(x)\right]=-\coth(x)\operatorname{csch}(x)$$

The six differentiation formulas are in the table below, written in chain rule form.

$$\frac{d}{dx}\left(\sinh(u)\right)=\cosh(u)\frac{du}{dx}$$
$$\frac{d}{dx}\left(\cosh(u)\right)=\sinh(u)\frac{du}{dx}$$
$$\frac{d}{dx}\left(\tanh(u)\right)=\operatorname{sech}^2(u)\frac{du}{dx}$$
$$\frac{d}{dx}\left(\coth(u)\right)=-\operatorname{csch}^2(u)\frac{du}{dx}$$
$$\frac{d}{dx}\left(\operatorname{sech}(u)\right)=-\operatorname{sech}(u)\tanh(u)\frac{du}{dx}$$
$$\frac{d}{dx}\left(\operatorname{csch}(u)\right)=-\operatorname{csch}(u)\coth(u)\frac{du}{dx}$$

Example: Differentiate: $y = \cosh(5x)$

deriv. of inside

$$y' = \sinh(5x) \cdot 5$$

Example: If $f(x) = \sinh\left(x^2 + 4x\right)$; find $f'(x) = ?$

$$f'(x) = \cosh(x^2 + 4x) \cdot [2x + 4]$$

$$f'(0) = \cosh(0) \cdot [2 \cdot 0 + 4] = 1 \cdot 4 = \boxed{4}$$

Example: Differentiate: $y = \tanh\left(x^3\right)$

$$y' = \operatorname{sech}^2(x^3) \cdot 3x^2$$

Example: Differentiate: $y = 5\operatorname{sech}\left(2e^x\right)$

$$y' = -5 \operatorname{sech}(2e^x) \cdot \tanh(2e^x) \cdot 2e^x$$

u'

Example: $f(x) = 5\sinh(2x)$; $f''(x) = ?$ $f^{(10)}(x) = ?$

$$f'(x) = 5 \cdot \cosh(2x) \cdot 2 \cdot$$

$$f''(x) = 5 \cdot \sinh(2x) \cdot 2 \cdot 2$$

$$\vdots \qquad 2 \cdot 2 \cdot 2$$

$$f^{(10)}(x) = 5 \cdot \boxed{\sinh(2x)} \cdot 2^{10}$$

Example: $f(x) = \ln\big(\underbrace{\sinh(5x)}_{u}\big)$; $f'(x) = ?$

$$f'(x) = \frac{u'}{u} = \frac{5 \cdot \cosh(5x)}{\sinh(5x)} = 5\coth(5x)$$

· product rule

Example: $\frac{d}{dx}\left[\underbrace{2\sinh(4x)}\underbrace{\cosh(4x)}\right]=$

$= 2\cdot\cosh(4x)\cdot 4\cdot\cosh(4x) + 2\sinh(4x)\cdot\sinh(4x)\cdot 4$

$= 8\cosh^2(4x) + 8\sinh^2(4x)$ //

$(u^3)' = 3u^2\cdot u'$

Example: Given $f(x)=\cosh(2x)+\sinh^3(x)$,

$f(0)=?$

$f(0) = \cosh(0) + \sinh^3(0) = 1+0=1$

$f'(x) = \sinh(2x)\cdot 2 + 3\cdot\sinh^2(x)\cdot\cosh(x)$

$f'(0)=?$

$f'(0) = 0\cdot 2 + 3\cdot 0^2\cdot 1 = \boxed{0}$

Example: Given $f(x) = \arctan\left(\overbrace{\sinh\left(4x^2\right)}^{u}\right)$, $f'(x) = ?$

$$f'(x) = \frac{u'}{1+u^2} = \frac{\cosh(4x^2) \cdot 8x}{1 + \sinh^2(4x^2)}$$

Exercise: $\frac{d}{dx}\left[\frac{\sinh(2x)}{1+\cosh(2x)}\right]=$

Exercise: $f(x)=\sinh(x+\cos(2x))$; $f'(0)=?$

Exercise: $f(x)=2\ln\left(\sinh(5x)\right)$; $f'(x)=?$

Exercise: $f(x)=\sqrt{1+\cosh(3x)}$; $f'(x)=?$

Exercise: $f(x)=\left(x+1\right)^{\cosh(x)}$; $f'(x)=?$ (Need to use logarithmic differentiation here).