Completed

Math 2413- Calculus I

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- Check CASA calendar for due dates.
- Bring "blank notes" to class. Completed notes will be posted after class.
- Do your best to attend every lecture and lab.
- Study after every lecture; work on the quiz covering the topic we cover on the lecture immediately afterwards. Retake your quizzes for more practice.
- Get help when you need help; bring your questions to the labs, or my office hours. We also have tutoring options on campus.
- Respect your friends in class; stay away from distractive behavior. Do your best to concentrate on the lecture.
- If you email me, mention the course code in the subject line. Email is the best way to communicate with me outside of class. Teams chat messages are not monitored or replied to.

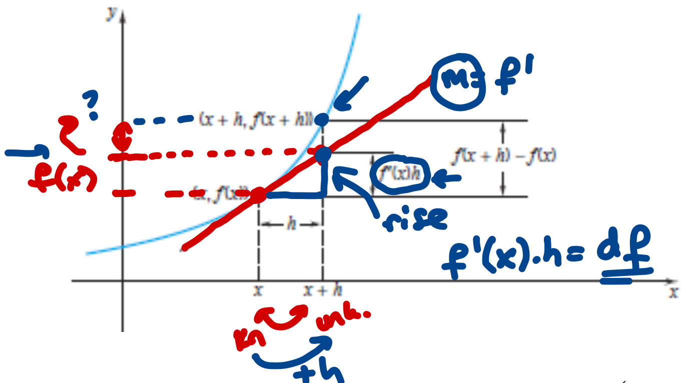
* Scheduler is open.
  Schedule your exam.

* WHW 10: 5.1 & 5.2
  WHW 11: 5.3
  ] Due Next week

* Take & retake online quizzes
  Ch5 quizzes are 19, 20 & 21.

$\sqrt{2}$

Section 5.2 - Differentials

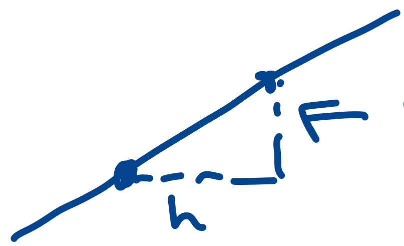


Definition: For $h \neq 0$, the difference $f(x+h) - f(x)$ is called the ***increment of*** f ***from*** x ***to*** $x+h$ and this increment is denoted by Δf:

$$\Delta f = f(x+h) - f(x).$$

The product $f'(x)h$ is called the ***differential*** f at x with increment h and is denoted by df:

$$\boxed{df = f'(x)h.}$$



dR

approx. change

For small h, $\Delta f \approx df$. Notice that

$$f(x+h) - f(x) \approx df \;\Rightarrow\; f(x+h) \approx f(x) + df.$$

To approximate $f(x+h)$, we first calculate the differential $df = f'(x)h$ and then add this to the function value:

$$\boxed{f(x+h) \approx f(x) + df.}$$

known

change

$f(x) = \sqrt{x} \rightarrow f'(x) = \frac{1}{2\sqrt{x}}$

$f(4) = 2 \leftarrow$ known

$(4, 2)$

$\sqrt{4.1} = ? \approx ?$

change

$\sqrt{4.1} \approx \sqrt{4} + \boxed{df}$

$df = \boxed{f'(4)} \cdot \underline{\underline{h}}$

$4 \xrightarrow[h=0.1]{} 4.1$

$df = f'(4) \cdot 0.1 = \frac{1}{4} \cdot 0.1 = \frac{1}{40}$

$$df = \frac{1}{40}$$

$$\sqrt{4.1} \approx \sqrt{4} + \frac{1}{40}$$

$$2 + \frac{1}{40} = \frac{81}{40}$$

Claim

$$\sqrt{4.1} \approx \frac{81}{40} = 2.025$$

$$\sqrt{4.1} = 2.02484\ldots$$

$df = ?$

$$f'(x) = \frac{1}{2\sqrt{x+5}}$$

Example 1: Find the differential of $f(x) = \sqrt{x+5}$ at $x = 4$ with increment $h = 0.1$.

$$df = f'(x) \cdot h = \underbrace{f'(4)} \cdot 0.1$$

$$= \frac{1}{2\sqrt{9}} \cdot \frac{1}{10}$$

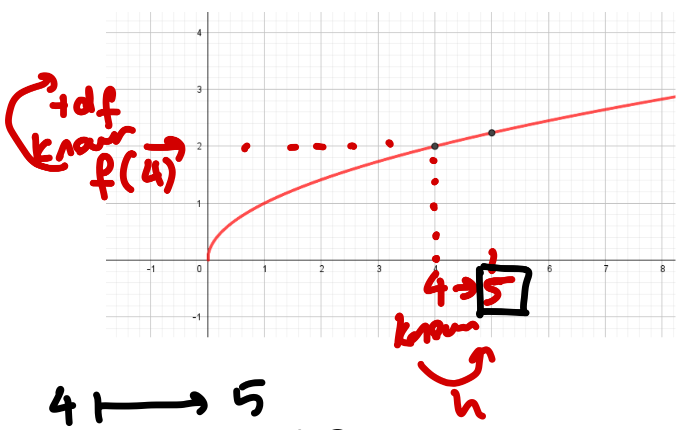
$$= \frac{1}{6} \cdot \frac{1}{10} = \boxed{\frac{1}{60}}$$

Example 2: Use differentials to approximate $\sqrt{5}$.

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$4 \xrightarrow[h=1]{} 5$



$4 \longmapsto 5$

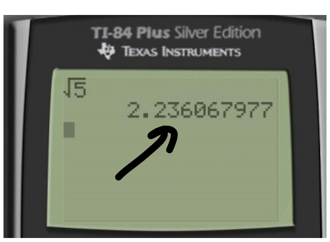
$\sqrt{4} \longmapsto \boxed{\sqrt{5}}$?

$\sqrt{5} \approx \sqrt{4} + df$

$df = f'(x) \cdot h$

$df = f'(4) \cdot 1$

$df = \frac{1}{2\sqrt{4}} \cdot 1 = \boxed{\frac{1}{4}}$



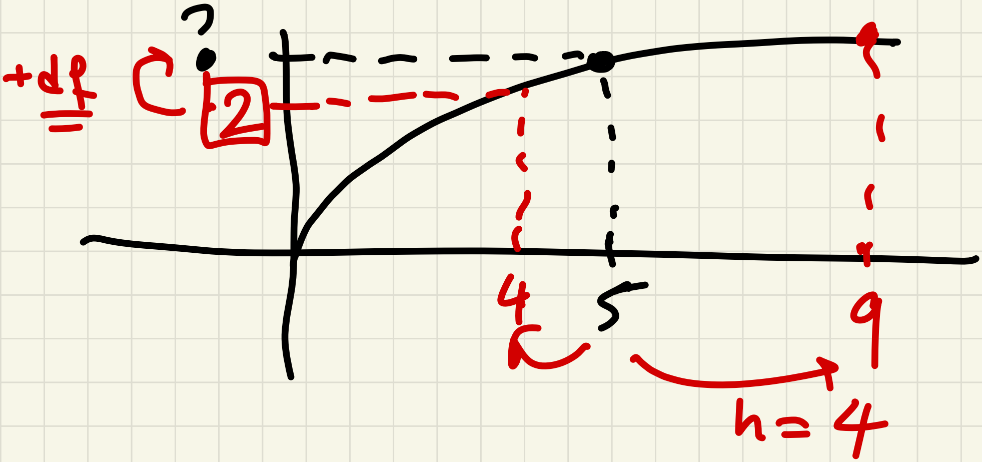
$\sqrt{5} \approx 2 + \frac{1}{4} = 2.25$

Claim: $\sqrt{5} \approx \boxed{2.25}$

small error

error: $2.25 - 2.236067977$

$\sqrt{5} = ?$

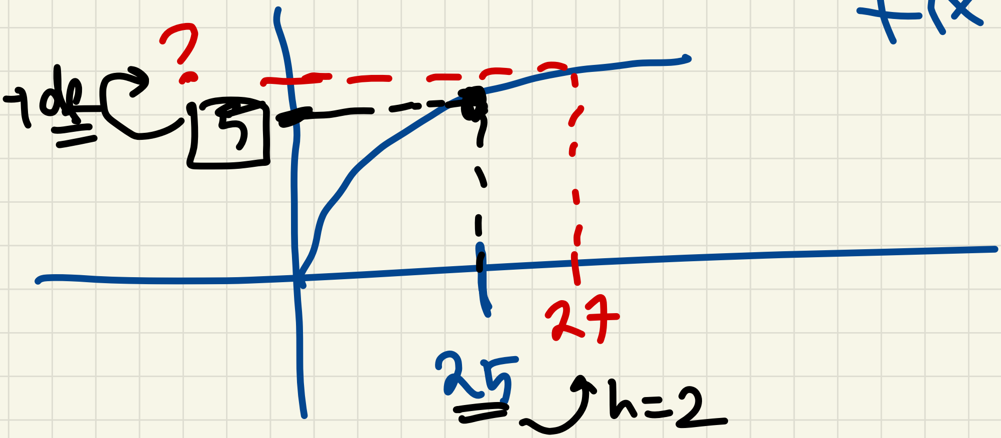


Smaller h is better

$\Rightarrow$ smaller error.

Approximate $\sqrt{27}$

$f(x) = \sqrt{x}$



$$\left[\, df = f'(x)\cdot h = f'(25)\cdot 2 = \frac{1}{2\sqrt{25}}\cdot 2 = \frac{1}{5} \right.$$

$$\sqrt{27} \approx \sqrt{25} + df$$

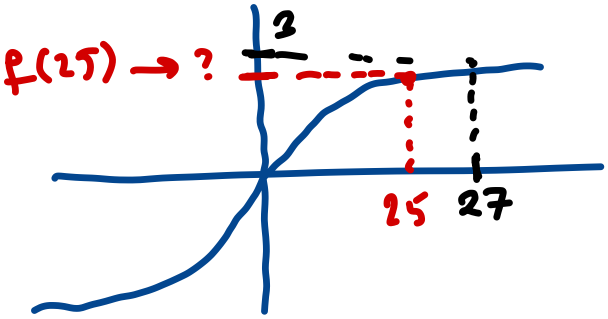
$$5 + \frac{1}{5} = 5.2$$

$$= \frac{26}{5}$$

$f(x) = \sqrt[3]{x} \Rightarrow f'(x) = \frac{1}{3} \cdot x^{-2/3}$

$(27)^{-2/3} = \frac{1}{(\sqrt[3]{27})^2}$

Example 3: Use differentials to approximate $\underbrace{\sqrt[3]{25}}$.



Reference point:

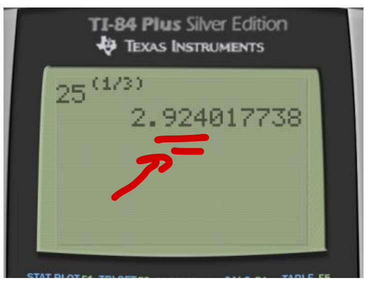
$\sqrt[3]{27} = 3$

$\underbrace{\boxed{27}}_{\text{ref.}} \xrightarrow[h=-2]{} \underbrace{25}_{\text{question}}$

$df = f'(x) \cdot h = f'(27) \cdot (-2)$

$df = \frac{1}{3}(27)^{-2/3} \cdot (-2) = \frac{1}{3} \cdot \frac{1}{9} \cdot -2$

$df = \boxed{\frac{-2}{27}}$



$\sqrt[3]{25} \approx \sqrt[3]{27} + df$

$3 + \frac{-2}{27} = \boxed{\frac{79}{27}}$

$\boxed{= 2.9259}$

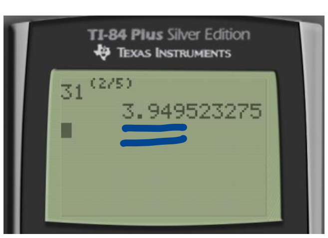
Example 4: Use differentials to approximate $31^{2/5}$.

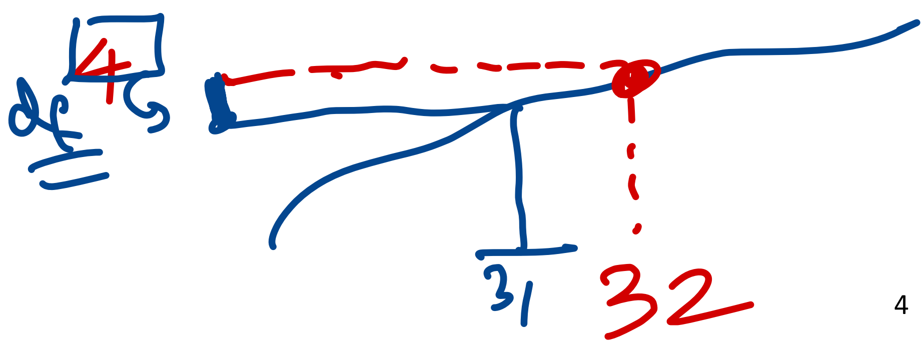
$31^{2/5}$

$f(x) = x^{2/5}$

Nice # to plug in

$f(32) = (32)^{2/5} = 4$





$(31)^{2/5} = ?$

$f(x) = x^{2/5}$ $\quad f(31) = ?$

$f'(x) = \frac{2}{5}(x)^{-3/5}$

$f'(32) = \frac{2}{5}(32)^{-3/5} = \frac{2}{5} \cdot \frac{1}{(\sqrt[5]{32})^3} = \frac{2}{5} \cdot \frac{1}{8}$

$= \frac{1}{20}$

$f(32) = (\sqrt[5]{32})^2 = (2)^2 = 4$

Known (reference point): $(32, 4)$

$\downarrow \quad \downarrow +df$

Question: $(31, ??)$

$32 \xrightarrow[h=-1]{} 31$

differential: $df = f'(32) \cdot h$

$df = \frac{1}{20} \cdot (-1)$

$df = -\frac{1}{20}$

$$f(31) \approx f(32) + df$$

$$4 + \frac{-1}{20}$$

$$= 4 - 0.05$$

$$= \boxed{3.95} \text{ or } \frac{79}{20}$$

$$(31)^{2/5} \approx \boxed{3.95}$$

our approximation

approx $\sqrt{35}$

$$f(x) = \sqrt{x}$$

Reference Point: $(36, 6)$

$$\begin{array}{ccc} & (36, & 6) \\ h=-1 & \downarrow & \downarrow df \\ & (35, & ?) \end{array}$$

$$df = f'(36) \cdot h_2$$

$$\sqrt{35} \approx \sqrt{36} + \boxed{df}$$

$f(x) = \ln(x)$ $\quad f'(x) = \frac{1}{x} \leftarrow f'(2) = \frac{1}{2}$

Example 5: Given that $\underbrace{\ln(2) \approx 0.69}$, use differentials to approximate $\boxed{\ln(2.1)}$.

$2 \xrightarrow[h=0.1]{} 2.1$

$(2, 0.69)$

$+0.1 \downarrow \quad \downarrow +df = \frac{1}{20}$

$(2.1, \boxed{??})$

$df = f'(2) \cdot h = \frac{1}{2} \cdot (0.1)$

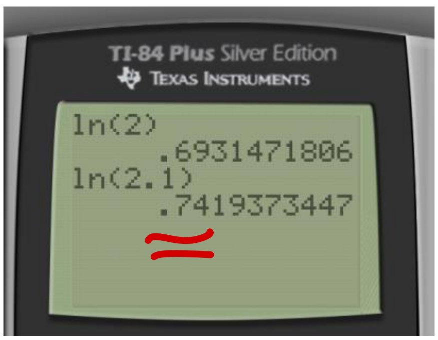
$= \frac{1}{20}$

$\ln(2.1) \approx \ln(2) + \frac{1}{20}$

$0.69 + 0.05$

$\boxed{0.74}$

our approx.



Example 5: Use differentials to approximate $\sin(29^0)$.

must use radians

convert to radians

$30^\circ \approx \frac{\pi}{6}$

$30^\circ \xrightarrow[-1^\circ]{} \boxed{29^\circ}$

$\frac{\pi}{6} \xrightarrow[h \; \boxed{-\frac{\pi}{180}}]{} \boxed{A}$

$df = f'\left(\frac{\pi}{6}\right) \cdot \left(-\frac{\pi}{180}\right)$

$\sin(A) \approx \underbrace{\sin(\pi/6)} + df$

$\frac{1}{2} + \boxed{df} = - - '$

In some problems, you're simply asked for the differential $df = f'(a) \cdot h$.

Example 6: The total cost incurred in operating a certain type of truck on a 500-mile trip, traveling at an average speed of x mph, is estimated to be $C(x) = 125 + x + \dfrac{4500}{x}$ dollars. Find the approximate change in total operating cost when the average speed is increased from 55mph to 58 mph.

df

$$55 \text{ mph} \xrightarrow[h=3]{} 58 \text{ mph}$$

$$df = f'(55) \cdot h$$

$$\text{Change} = \left(1 - \frac{4550}{(55)^2}\right) \cdot 3$$

$f(x) = \sqrt{x}$ $\quad f(4)$

$(4, 2)$

$h=2 \quad +df$

$(6, \boxed{???})$

$f(6) = \sqrt{6}$

Attendance Popper #

Q# Use differentials to approximate $\sqrt{6}$.

a) 7/4
b) 5/2
c) 3
d) 21/8
e) None

Q# Find the differential of $f(x) = \sqrt{x}$ at x=9 with increment h = 0.3.

a) 1/10
b) 1/5
c) 1/20
d) 2/15
e) None

Q# Use differentials to approximate $(33)^{1/5}$.

a) 163/80 b) 161/80 c) 81/40 d) 83/40 e) None