

Math 2413- Calculus I

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- Check CASA calendar for due dates.
- Bring “blank notes” to class. Completed notes will be posted after class.
- Do your best to attend every lecture and lab.
- Study after every lecture; work on the quiz covering the topic we cover on the lecture immediately afterwards. Retake your quizzes for more practice.
- Get help when you need help; bring your questions to the labs, or my office hours. We also have tutoring options on campus.
- Respect your friends in class; stay away from distractive behavior. Do your best to concentrate on the lecture.
- **If you email me, mention the course code in the subject line. Email is the best way to communicate with me outside of class. Teams chat messages are not monitored or replied to.**

Chapter 6 – Integration

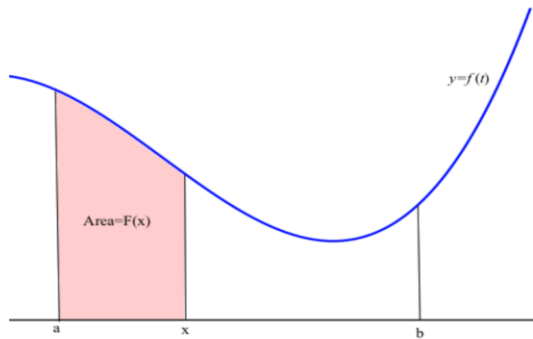
Section 6.2 - The Fundamental Theorem of Calculus

Let f be a continuous function over the interval $[a, b]$. Define a new function by

$$F(x) = \int_a^x f(t) \, dt.$$

Here, the upper limit x varies between a and b .

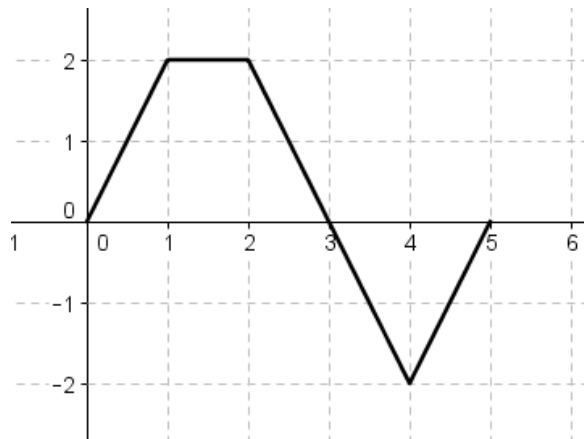
If f happens to be a nonnegative function, then $F(x)$ can be seen as the “area under the graph of f from a to x ”. We can think of $F(x)$ as the “accumulated area” function.



Visual: <https://www.geogebra.org/m/fhmxyw9>

Example: If f is the function whose graph is given below, and $F(x) = \int_0^x f(t) \, dt$,

find the values: $F(0)$, $F(3)$ and $F(5)$.



Theorem: Fundamental Theorem of Calculus Part 1

If f is a continuous function over the interval $[a, b]$, then the function

$$F(x) = \int_a^x f(t) \, dt$$

is continuous on $[a, b]$ and differentiable on (a, b) . Moreover,

$$F'(x) = \frac{d}{dx} \int_a^x f(t) \, dt = f(x), \text{ for all } x \text{ in } (a, b).$$

Ex: If you define a function: $F(x) = \int_a^x (t^2) \, dt$, by integrating $f(t) = t^2$, we know the derivative of this function: $F'(x) = x^2$

Example: $F(x) = \int_1^x (t^2 + 2t) \, dt, \quad F'(x) =$

Example: $F(x) = \int_{\pi}^x 5 \cos(2t) \, dt, \quad F'(4\pi) =$

Example: $F(x) = \int_x^1 \sin(2t) \, dt, \quad F'(x) =$

Remark: If the upper boundary is an expression involving x :

$$\frac{d}{dx} \left(\int_a^{u(x)} f(t) \, dt \right) = f(u(x)) u'(x).$$

Example: $F(x) = \int_1^{5x^2} \frac{1}{1+t^2} \, dt, \quad F'(x) =$

What if both boundaries are expressions involving x?

Remark: $\frac{d}{dx} \left(\int_{u(x)}^{v(x)} f(t) dt \right) = f(v(x))v'(x) - f(u(x))u'(x)$

Ex: $F(x) = \int_{6x}^{x^2} f(t) dt \rightarrow F'(x) = f(x^2) \cdot 2x - f(6x) \cdot 6$

Example: $F(x) = \int_{x^2}^{2x^3} (5t^2) dt, \quad F'(x) =$

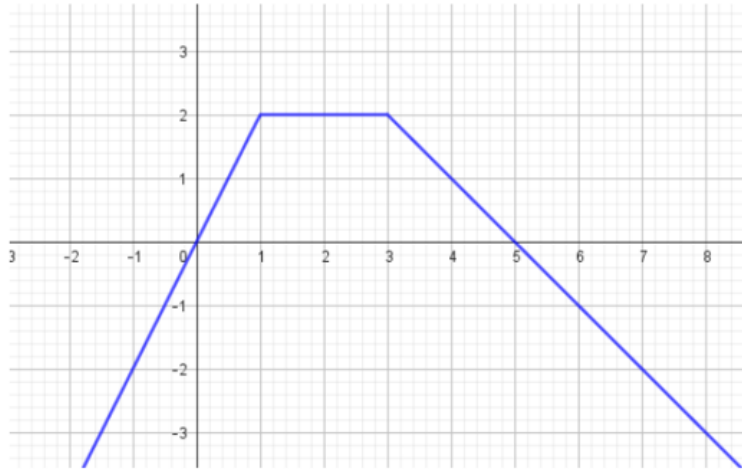
Exercise: $\frac{d}{dx} \left[\int_{\sin x}^{2x} (1 + \sqrt{t}) dt \right],$

Example: Let f be a continuous function satisfying $x^3 + x^2 - x = \int_1^x f(t) \, dt$.

Find $f(x)$.

Find $f'(x)$.

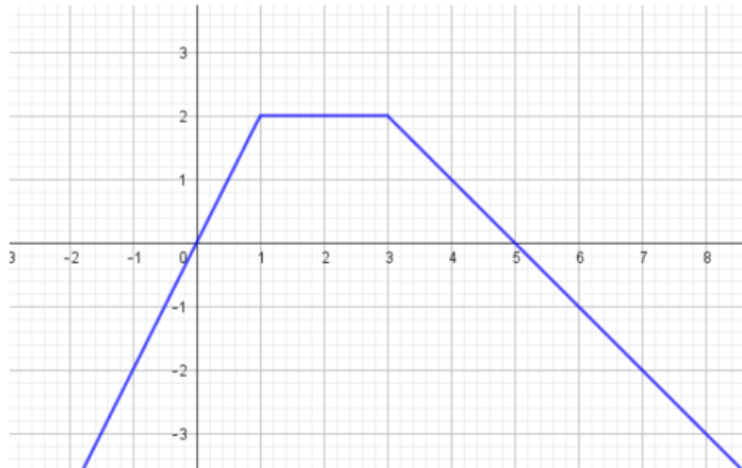
Exercise: The graph of $f(x)$ is given below. If $F(x) = \int_0^x f(t)dt$, find $F'(3) = ?$



Exercise: The graph of $f(x)$ is given below.

$$\int_0^2 f(x)dx + \int_0^3 f'(x)dx = ?$$

$$\int_3^5 (f(x) + f'(x))dx = ?$$



HOMEWORK: READ Section 6.2 from your text book! Study the examples there.

Exercise: For $x > 1$, let $F(x) = \int_1^x (t^2 - 6t) dt$.

When is F increasing?

Find the critical numbers of F .

Study concavity of F .

Summary-

If we consider an integral as an “accumulation of area”, then the derivative of the integral is a “rate of change” of an “accumulation of an area”.

Therefore, if $F(x) = \int_a^x f(t) dt$, then $F'(x) = f(x)$.

- $F(a) = 0$
- Where f is positive, F is increasing
- Where f is negative, F is decreasing
- Where f is zero, F has possible max, min or inflection point
- Where f is increasing, F is concave up
- Where f is decreasing, F is concave down

Next: Fundamental Theorem of Calculus Part 2

First, what is an “anti-derivative”?

Definition: Let f be a continuous function over the interval $[a,b]$. A function G is called an **anti-derivative for** f over the interval $[a,b]$ if G is continuous on $[a,b]$ and $G'(x) = f(x)$ for all x in (a,b) .

That is, an antiderivative of a function $f(x)$ is a function $G(x)$ whose derivative is $f(x)$.

Question: Why “an”, not “the”?

Ex: $f(x) = 2x$; an antiderivative for this function is: $G(x) =$

Ex: $f(x) = 3x^2$; an antiderivative for this function is: $G(x) =$

Ex: $f(x) = \frac{1}{x}$; an antiderivative for this function is: $G(x) =$

Ex: $f(x) = \cos(x)$; an antiderivative for this function is: $G(x) =$

Theorem: Fundamental Theorem of Calculus Part 2

Let f be a continuous function over the interval $[a, b]$. If G is any antiderivative for f over the interval $[a, b]$, then

$$\int_a^b f(x) \, dx = G(b) - G(a).$$

Ex: To compute: $\int_1^5 (2x) \, dx$; we first find an anti-derivative of $f(x) = 2x$;

$$G(x) = x^2$$

$$\int_1^5 (2x) \, dx = \left[x^2 \right]_1^5 = (5^2) - (1^2) = 25 - 1 = 24$$

Ex: To compute: $\int_1^2 (3x^2) \, dx$; we first find an anti-derivative of $f(x) = 3x^2$;

$$G(x) = x^3$$

$$\int_1^2 (3x^2) \, dx = \left[x^3 \right]_1^2 = (2^3) - (1^3) = 8 - 1 = 7$$

Ex: To compute: $\int_0^1 (5x^4) \, dx$; we first find an anti-derivative of $f(x) = 5x^4$;

$$G(x) = x^5$$

$$\int_0^1 (5x^4) \, dx = \left[x^5 \right]_0^1 = (1^5) - (0^5) = 1 - 0 = 1$$

Example: Calculate the definite integral using FTC.

$$\int_1^2 (5x^4 + 3x^2 + 2x) \, dx =$$

Example: Calculate the definite integral using FTC.

$$\int_1^4 \frac{1}{x} \, dx =$$

Example: Calculate the definite integral using FTC.

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos(x) dx =$$

Next: in Section 6.3, we will learn some rules to make finding the anti-derivative easier.