Math 2413- Calculus I

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- Check CASA calendar for due dates.
- Bring "blank notes" to class. Completed notes will be posted after class.
- Do your best to attend every lecture and lab.
- Study after every lecture; work on the quiz covering the topic we cover on the lecture immediately afterwards. Retake your quizzes for more practice.
- Get help when you need help; bring your questions to the labs, or my office hours. We also have tutoring options on campus.
- Respect your friends in class; stay away from distractive behavior. Do your best to concentrate on the lecture.
- **If you email me, mention the course code in the subject line. Email is the best way to communicate with me outside of class. Teams chat messages are not monitored or replied to**.

Section 6.3 – Basic Integration Rules

To understand Section 6.4; you must know 6.3 very well!!!

TABLE OF INTEGRALS

$\int x^r \, dx = \dfrac{x^{r+1}}{r+1} + C \, ; \; r \neq -1$

$\int \dfrac{1}{x} \, dx = ln|x| + C$

$\int sin\, x \, dx = -cos\, x + C$

$\int cos\, x \, dx = sin\, x + C$

$\int sec^2 x \, dx = tan\, x + C$

$\int csc^2 x \, dx = -cot\, x + C$

$\int sec\, x\, tan\, x \, dx = sec\, x + C$

$\int csc\, x\, cot\, x \, dx = -csc\, x + C$

$\int e^x \, dx = e^x + C$

$\int a^x \, dx = \dfrac{a^x}{ln\, a} + C \; ; \; a > 0, \; a \neq 1.$

$\int sinh\, x \, dx = cosh\, x + C$

$\int cosh\, x \, dx = sinh\, x + C$

$\int \dfrac{1}{\sqrt{1-x^2}} \, dx = arcsin\, x + C$

$\int \dfrac{1}{1+x^2} \, dx = arctan\, x + C$

Question# **)** $\int\left(x^3+2x+\frac{1}{x}\right)dx=?$

a) $x^4+x^2+\ln(x)+C$

b) $\frac{x^4}{4}+x^2+\ln|x|+C$

c) $\frac{x^4}{4}+x^2-\ln|x|+C$

d) **None**

Question# **)** $\int\left(\sqrt{x}+\frac{1}{x^2}\right)dx=?$

a) $x^{3/2}+\ln(x)+C$

b) $\frac{3x^{3/2}}{2}+\frac{1}{x}+C$

c) $\frac{2x^{3/2}}{3}-\frac{1}{x}+C$

d) $\frac{2x^{3/2}}{3}+\frac{1}{x}+C$

e) **None**

Question# **)** $\int\left(e^x+\sin x\right)dx=?$

a) $e^x+\cos x+C$

b) $e^x-\cos x+C$

c) $e^x-\sin x+C$

d) **None**

Question#   ) $\int (2x+5)dx = ?$

- **a)** x^2+5x+C
- **b)** x^2-5x+C
- **c)** $3x^2+C$
- **d)** **None**

Question#   ) $\int (2x+5)^2\, dx = ?$

- **a)** x^2+5x+C
- **b)** $\dfrac{(2x+5)^3}{3}+C$
- **c)** $\dfrac{4x^3}{3}+10x^2+25x+C$
- **d)** $4x^3+10x^2+25x+C$
- **e)** **None**

What if we need to compute: $\int (2x+5)^3\, dx = ?$ **or** $\int (2x+5)^{10}\, dx = ?$

Section 6.4 – Integration by Substitution

The **method of substitution** is based on the Chain Rule:

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x).$$

When we have an integrand of the form $f'(g(x)) \cdot g'(x)$, we can find the integral by reversing this process:

$$\int f'(g(x)) \cdot g'(x)\ dx = (f \circ g)(x).$$

In the integral, we let $u = g(x)$ and then the differential of u is $du = g'(x)dx$. Hence,

.

Replacing $g(x)$ by u and $g'(x)dx$ by du is called **substitution (**or u – **substitution**). We use this method for when the integrand is a composition of functions.

Example: Evaluate the integral:

$$\int 2x\left(x^2+5\right)^3 dx$$

Example: Evaluate the integral:

$$\int (2x+3)\left(x^2+3x\right)^5 dx$$

Example: Evaluate the integral:

$$\int \frac{10x}{\left(x^2+2\right)^4} dx$$

Example: Evaluate the integral:

$$\int x\sqrt{x^2+12}\,dx$$

Example: Evaluate the integral:

$$\int \frac{8x^3}{\sqrt{x^4+2}}\,dx$$

Example: Evaluate the integral:

$$\int 2\cos x\sqrt{4+\sin x}\,dx$$

Exercise: Evaluate the integral: $\int \frac{\cos x}{\sqrt{4+\sin x}}\,dx$

Exercise: Evaluate the integral: $\int \frac{e^x}{\sqrt{4+e^x}}\,dx$

Integrals of Exponential Functions: **Formula:** $\int e^u du = e^u + C$

Example: $\int e^{4x} dx$

MUST KNOW: $\int e^{mx} dx = \dfrac{e^{mx}}{m} + C$

Exercise: $\int e^{12x} dx =$

Example: $\int xe^{x^2+1}dx$

Example: $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx =$

Trigonometric Functions

Example: $\int \cos(4x)\,dx = ?$

Formula: $\int \cos(u)du = \sin(u) + C$

MUST KNOW:

$$\int \sin(mx)dx = \frac{-\cos(mx)}{m} + C$$

$$\int \cos(mx)dx = \frac{\sin(mx)}{m} + C$$

Example: $\int x\sin\left(5x^2\right)dx = ?$

Formula: $\int \sin(u)du = -\cos(u) + C$

Example: $\int e^x \sec^2\left(e^x\right)dx = ?$

Formula: $\int \sec^2(u)du = \tan(u) + C$

Products of Trigonometric Functions

Example: Evaluate the integral:

$$\int \sin(x)\cos(x)\,dx$$

Example: Evaluate the integral:

$$\int \cos^6(x)\sin(x)\,dx$$

Example: Evaluate the integral:

$$\int \sec^2(x)\tan(x)\,dx$$

Some rational functions (special case!):

Example: Evaluate: $\int \frac{6x}{x^2+5}dx$

Example: Evaluate: $\int \frac{2x+6}{x^2+6x+1}dx$

Example: Evaluate: $\int \frac{e^x}{5e^x + 1} dx$

Example: Evaluate: $\int \frac{8}{4x+1} dx$

Exercise:

$$\int \frac{\sin(x)}{2+\cos(x)}\,dx = ?$$

$$\int \frac{1}{2+x} + \frac{1}{x-1} + \frac{1}{x^2}\,dx = ?$$

$$\int \frac{1}{(x-1)^2}\,dx = ?$$

The method of u-substitution with <u>Definite Integrals</u>

You either integrate and recall the limits after finding the integral in terms of x,

OR

change the limits of integration (but fund the integral only in terms of u, not x!)

Example:

OPTION 1:

$$\int_0^1 3x^2\left(x^3+2\right)^3\ dx \xrightarrow[\substack{x=0\to u=2\\ x=1\to u=3}]{u=x^3+2,du=3x^2dx} \int_2^3 u^3du = \left[\frac{u^4}{4}\right]_2^3 = \frac{3^4}{4}-\frac{2^4}{4} = \frac{65}{4}$$

OPTION 2:

$$\int_0^1 3x^2\left(x^3+2\right)^3\ dx \xrightarrow{u=x^3+2;du=3x^2dx} \int u^3du = \left[\frac{u^4}{4}+C\right] \xrightarrow{back\ to\ "x"} \left[\frac{(x^3+2)^4}{4}+C\right]_0^1$$

$$=\frac{3^4}{4}-\frac{2^4}{4}=\frac{65}{4}$$

Example: Evaluate:

$$\int_0^2 \frac{4x}{x^2+1}\, dx =$$

Example: Evaluate:

$$\int_0^{\pi/2} \frac{\cos x}{2+\sin x}\, dx =$$

Here is a summary of the formulas we covered in this section:

TABLE OF INTEGRALS

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + C, \text{ for } n \neq -1.$$

$$\int \frac{1}{u} \, du = ln|u| + C \qquad \text{and} \qquad \int \frac{g'(x)}{g(x)} dx = ln|g(x)| + C$$

$$\int tan\, x \, dx = -ln|cos\, x| + C = ln|sec\, x| + C$$

$$\int sec\, x \, dx = ln|sec\, x + tan\, x| + C$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = arcsin\left(\frac{x}{a}\right) + C \qquad \int \frac{1}{\sqrt{a^2 - u^2}} \, du = arcsin\left(\frac{u}{a}\right) + C$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} arctan\left(\frac{x}{a}\right) + C \qquad \int \frac{1}{a^2 + u^2} \, du = \frac{1}{a} arctan\left(\frac{u}{a}\right) + C$$

Homework: Read Section 6.4 from your text book to see how these formulas are derived. Study the examples there.

Study these solved examples (applications of formulas). These type of integrals will show up a lot in Calculus 2.

FORMULA: $\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right)+C$

Ex: $\int \frac{1}{4+x^2}\,dx = \frac{1}{2}\arctan\left(\frac{x}{2}\right)+C$

Ex: $\int \frac{2}{9+x^2}\,dx = 2\int \frac{1}{9+x^2}\,dx = \frac{2}{3}\arctan\left(\frac{x}{3}\right)+C$

Ex:

$$\int \frac{2}{9+25x^2}\,dx = 2\int \frac{1}{9+(5x)^2}\,dx \xrightarrow[du=5dx]{u=5x} 2\int \frac{1}{9+u^2}\cdot\frac{du}{5} = \frac{2}{5}\cdot\frac{1}{3}\arctan\left(\frac{u}{3}\right)+C$$

$$= \frac{2}{15}\arctan\left(\frac{5x}{3}\right)+C$$

FORMULA: $\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \arcsin\left(\frac{x}{a}\right) + C$

Ex: $\int \frac{1}{\sqrt{4 - x^2}}\,dx = \arcsin\left(\frac{x}{2}\right) + C$

Ex: $\int \frac{2}{\sqrt{9 - x^2}}\,dx = 2\int \frac{1}{\sqrt{9 - x^2}}\,dx = 2\arcsin\left(\frac{x}{3}\right) + C$

Ex:

$$\int \frac{2}{\sqrt{9 - 49x^2}}\,dx = \int \frac{2}{\sqrt{9 - (7x)^2}}\,dx \xrightarrow[du=7dx]{u=7x} \frac{2}{7}\int \frac{1}{\sqrt{9 - u^2}}\,du = \frac{2}{7}\arcsin\left(\frac{u}{3}\right) + C$$

$$= \frac{2}{7}\arcsin\left(\frac{7x}{3}\right) + C$$

NOTE: The integrals $\int \frac{x}{\sqrt{4 - x^2}}\,dx$ or $\int \frac{x}{4 + x^2}\,dx$ are NOT about these formulas: they are typical “u-sub” questions. Do you see why?

Remark: Make sure you know how to integrate the following: (Very important!)

$$\int \frac{ax}{x^2+c}dx$$

$$\int \frac{b}{x^2+c}dx$$

$$\int \frac{ax+b}{x^2+c}dx$$

MISCALLENAUS EXERCISES:

Compute the following integrals:

1) $\int \frac{2x}{x^2+5}dx$

2) $\int \frac{4x+1}{x^2+1}dx$

3) $\int \frac{4}{x+1}+\frac{2}{x-1}dx$

4) $\int \frac{4x+8}{\sqrt{x^2+4x+1}}\ dx$

5) $\int_0^1 \frac{6x+12}{\sqrt{x^2+4x+1}}dx =$

6) $\int_0^1 x\left(x^2+1\right)^3 dx =$

7) $\int \frac{e^x}{e^{2x}+4}dx$

8) $\int \frac{e^x}{e^x+4}dx$

9) $\int \frac{4x+8}{x^2+4x+1}dx$

10) $$\int_0^{\pi} \sin(x)\sqrt{8+\cos(x)}\,dx =$$

11) $$\int_0^{\pi/2} \sin(x)\cos^2(x)\,dx =$$

12) $$\int \sin(2x)+\cos(4x)\,dx =$$

13) $$\int \left(e^{5x}+\frac{1}{x^2}\right)dx =$$

14) $$\int \frac{1}{(x+1)^2}\,dx = ?$$

15) $$\int \frac{1}{(2x+1)^2}\,dx = ?$$