Math 2413- Calculus I

Dr. Melahat Almus

Email: malmus@uh.edu

completed

- Check CASA calendar for due dates.
- Bring "blank notes" to class. Completed notes will be posted after class.
- Do your best to attend every lecture and lab.
- Study after every lecture; work on the quiz covering the topic we cover on the lecture immediately afterwards. Retake your quizzes for more practice.
- Get help when you need help; bring your questions to the labs, or my office hours. We also have tutoring options on campus.
- Respect your friends in class; stay away from distractive behavior. Do your best to concentrate on the lecture.
- **If you email me, mention the course code in the subject line. Email is the best way to communicate with me outside of class. Teams chat messages are not monitored or replied to**.

Section 6.3 – Basic Integration Rules

To understand Section 6.4; you must know 6.3 very well!!!

TABLE OF INTEGRALS

$$\int x^r \, dx = \frac{x^{r+1}}{r+1} + C \, ; \; r \neq -1$$

$$\int \frac{1}{x} \, dx = ln|x| + C$$

$$\int sin\, x \, dx = -cos\, x + C$$

$$\int cos\, x \, dx = sin\, x + C$$

$$\int sec^2 x \, dx = tan\, x + C$$

$$\int csc^2 x \, dx = -cot\, x + C$$

$$\int sec\, x\, tan\, x \, dx = sec\, x + C$$

$$\int csc\, x\, cot\, x \, dx = -csc\, x + C$$

$$\int e^x \, dx = e^x + C$$

$$\int a^x \, dx = \frac{a^x}{ln\, a} + C \; ; \; a > 0, \; a \neq 1.$$

$$\int sinh\, x \, dx = cosh\, x + C$$

$$\int cosh\, x \, dx = sinh\, x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = arcsin\, x + C$$

$$\int \frac{1}{1+x^2} \, dx = arctan\, x + C$$

Question#) $\int\left(x^3+2x+\frac{1}{x}\right)dx=?$

a) $x^4+x^2+\ln(x)+C$

b) $\frac{x^4}{4}+x^2+\ln|x|+C$

c) $\frac{x^4}{4}+x^2-\ln|x|+C$

d) None

6.3

Question#) $\int\left(\sqrt{x}+\frac{1}{x^2}\right)dx=?$

a) $x^{3/2}+\ln(x)+C$

b) $\frac{3x^{3/2}}{2}+\frac{1}{x}+C$

c) $\frac{2x^{3/2}}{3}-\frac{1}{x}+C$

d) $\frac{2x^{3/2}}{3}+\frac{1}{x}+C$

e) None

6.3

Question#) $\int\left(e^x+\sin x\right)dx=?$

a) $e^x+\cos x+C$

b) $e^x-\cos x+C$

c) $e^x-\sin x+C$

d) None

6.3

Question# **)** $\int (2x+5)dx = ?$

a) $x^2 + 5x + C$
b) $x^2 - 5x + C$
c) $3x^2 + C$
d) None

$\rightarrow \otimes^{\text{number}}$

Question# **)** $\int (2x+5)^2\, dx = ?$

$\int (\text{ugly})^{\text{number}}\, dx$

a) $x^2 + 5x + C$
b) $\frac{(2x+5)^3}{3} + C$
c) $\frac{4x^3}{3} + 10x^2 + 25x + C$
d) $4x^3 + 10x^2 + 25x + C$
e) None

What if we need to compute: $\int (2x+5)^3\, dx = ?$ **or** $\int (2x+5)^{10}\, dx = ?$

$$\int \frac{x^2}{\sqrt{x^3+4}}\, dx$$

$$x^3 \xrightarrow{\text{deriv}} 3x^2$$

anti-deriv

int

$\sum$

$$(x^2+5)^4 \xrightarrow{\text{deriv.}} 4(x^2+5)^3 \cdot 2x$$

$$\boxed{8x(x^2+5)^3}$$

anti-deriv.

Section 6.4 – Integration by Substitution

The **method of substitution** is based on the Chain Rule:

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x).$$

When we have an integrand of the form $f'(g(x)) \cdot g'(x)$, we can find the integral by reversing this process:

$$\int f'(g(x)) \cdot g'(x)\ dx = (f \circ g)(x).$$

In the integral, we let $u = g(x)$ and then the differential of u is $du = g'(x)dx$. Hence,

.

Replacing $g(x)$ by u and $g'(x)dx$ by du is called **substitution (**or u – **substitution**). We use this method for when the integrand is a composition of functions.

case-1

(ugly expression)$^{\text{number}}$

Example: Evaluate the integral:

$$\int 2x\left(x^2+5\right)^3 dx \overset{\text{u-sub}}{=} \int u^3 \cdot du \overset{\text{power rule}}{=} \frac{u^4}{4} + C$$

back to x

$$= \boxed{\frac{(x^2+5)^4}{4} + C}$$

deriv.

$$\frac{1}{\cancel{4}} \cancel{4}(x^2+5)^3 \cdot 2x$$

chain rule

Let $u = x^2 + 5$

deriv.

$$\frac{du}{dx} = 2x$$

$$\Rightarrow du = \boxed{2x \cdot dx}$$

Day 28 - §6.4 continued

Power rule

$$\int x^r \, dx = \frac{x^{r+1}}{r+1} + C$$

$(r \neq -1)$

$$\int x^2 \, dx = \frac{x^3}{3} + C$$

$$\int t^2 \cdot \underline{\underline{dt}} = \frac{t^3}{3} + C$$

$$\int u^2 \cdot d\underline{\underline{u}} = \frac{u^3}{3} + C$$

$$\int (x^3+1)^2\,dx \cancel{=} \frac{(x^3+1)^3}{3} + C$$

$$(\text{ugly})^{\text{number}}$$

$$\int 2x\,(x^2+1)^3\,dx = \int u^3 \cdot du$$

$$u = x^2+1$$

$$\downarrow$$

$$\frac{du}{dx} = 2x$$

$$du = \underbrace{2x\,dx}$$

$$= \frac{u^4}{4} + C$$

$$\downarrow$$

$$= \frac{(x^2+1)^4}{4} + C$$

$$\int (x^2+1)^3\,dx$$

~~u-sub~~

$$u = x^2+1$$

$$du = 2x\,dx$$

?

power rule

Example: Evaluate the integral:

$$\int(2x+3)(x^2+3x)^5\,dx = \int u^5 \cdot du = \frac{u^6}{6}+C$$

match !!!

Let $u = x^2+3x$

$$\frac{du}{dx} = 2x+3$$

$$du = \boxed{(2x+3)\cdot dx}$$

???

$$= \boxed{\frac{(x^2+3x)^6}{6}+C}$$

Example: Evaluate the integral:

$$\int\frac{10x}{(x^2+2)^4}dx = \int\frac{5\cdot 2x\cdot dx}{(x^2+2)^4}$$

switch

$u = x^2+2$

$$\frac{du}{dx} = 2x$$

$$du = \boxed{2x\,dx}$$

$$= \int\frac{5}{u^4}\cdot du$$

$$= 5\cdot\int u^{-4}\cdot du$$

power rule

$$= 5\cdot\frac{u^{-3}}{-3}+C$$

$= -\frac{5}{3}(x^2+2)^{-3} + C$

final answer.

6.3

$\int \sqrt{x}\, dx = \int x^{1/2} dx = \frac{x^{3/2}}{3/2} + C$

$\int \sqrt{5x+1}\, dx$ ~~copy~~ ??

$1/2 + 1 = 3/2$

$\int \sqrt{u}\, du = \int u^{1/2} du = \frac{u^{3/2}}{3/2} + C$

$\sqrt{\text{ugly}}$ generally calls for u-sub.

Example: Evaluate the integral: Try u = inside

$$\int x\sqrt{x^2+12}\,dx = \int \sqrt{u}\cdot\frac{du}{2} = \frac{1}{2}\int\sqrt{u}\cdot du$$

$u = x^2+12$

$\frac{du}{dx} = 2\cdot x$

$du = 2\cdot \boxed{x\cdot dx}$

$x\cdot dx = \frac{du}{2}$

power rule

$$= \frac{1}{2}\cdot\frac{u^{3/2}}{3/2}+C$$

back to x

$$= \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}(x^2+12)^{3/2}+C$$

$$= \boxed{\frac{1}{3}(x^2+12)^{3/2}+C}$$

$2\cdot 4x^3$

Example: Evaluate the integral:

$$\int\frac{8x^3}{\sqrt{x^4+2}}dx = \int\frac{2}{\sqrt{u}}\cdot du$$

6.3

$u = x^4+2$

$\frac{du}{dx} = 4x^3$

$du = 4x^3\cdot dx$

$2du = 8x^3dx$

$$= 2\cdot\int u^{-1/2}\cdot du$$

power rule

$$= 2\cdot\frac{u^{1/2}}{1/2}+C$$

back to x

$$= \boxed{4\sqrt{x^4+2}+C}$$

Check

$4 \cdot \sqrt{x^4+2} + C$

$\downarrow$ derivative

$\cancel{4}^{\,2} \cdot \dfrac{4 \cdot x^3}{\cancel{2}\sqrt{x^4+2}} + 0$

$\sqrt{\text{ugly}}$

$\dfrac{u'}{2\sqrt{u}}$ ← chain rule

$=$

$\boxed{\dfrac{8x^3}{\sqrt{x^4+2}}}$ ← integrand

$\sqrt{\text{ugly}}$ $\quad u = \text{inside}$

Example: Evaluate the integral:

$\int 2\cos x\sqrt{4+\sin x}\,dx = 2\cdot\int\sqrt{u}\cdot du = 2\cdot\int u^{1/2}du$

$= 2\cdot\dfrac{u^{3/2}}{3/2}+C$

$= \dfrac{4}{3}(4+\sin(x))^{3/2}+C$

$u = 4+\sin(x)$

$\dfrac{du}{dx} = \cos(x)$

$du = \cos(x)\cdot dx$

Exercise: Evaluate the integral: $\int \frac{\cos x}{\sqrt{4+\sin x}}dx$

Exercise: Evaluate the integral: $\int \frac{e^x}{\sqrt{4+e^x}}dx$

$(e^{x^2+1})'$

$\int e^{x^2+1}\,dx \cancel{= e^{x^2+1}} + C$

case-2

6.3 $\int e^x\,dx = e^x + C$

$\int e^u\,du = e^u + C$

Integrals of Exponential Functions:

Formula: $\int e^u\,du = e^u + C$

u = exponent

Example: $\int e^{4x}dx$

$= \int e^u \cdot \frac{du}{4} = \frac{1}{4}\int e^u \cdot du$

↓ formula

$= \frac{1}{4}e^u + C$

back to x

$= \boxed{\frac{1}{4}e^{4x} + C}$

$u = 4x$

$\frac{du}{dx} = 4$

$du = 4\,\boxed{dx}$

$dx = \frac{du}{4}$

MUST KNOW: $\int e^{mx}dx = \frac{e^{mx}}{m} + C$

Exercise: $\int e^{12x}dx =$

$\frac{1}{12} \cdot e^{12x} + C$

what if? $\int e^{x^2+1}\,dx \neq \int e^u \cdot \frac{du}{2x} = \frac{1}{2x}\int e^u du$

e^{ugly} u: exponent

Example: $\int xe^{x^2+1}dx$ $= \int e^u \cdot \frac{du}{2} = \frac{1}{2}\int e^u\, du$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$du = 2.\boxed{x.dx}$

$x.dx = \frac{du}{2}$

formula

$= \frac{1}{2}.e^u + c$

back to x

$= \frac{1}{2}.e^{x^2+1} + c$

calc2

Example: $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx =$ $\int e^{\sqrt{x}} \cdot \frac{dx}{\sqrt{x}} = \int e^u \cdot 2du$

$u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$du = \frac{1}{2\sqrt{x}}.dx$

$2.du = \frac{dx}{\sqrt{x}}$

$= 2\int e^u\, du$

$= 2e^u + c$

$\boxed{= 2e^{\sqrt{x}} + c}$

Note:

$$\int a^x\,dx = \frac{a^x}{\ln(a)} + C$$

$$\int 2^x\,dx = \frac{2^x}{\ln(2)} + C$$

$$\int 2^{3x+1}\,dx = \frac{1}{3}\int 2^u \cdot du$$

$u = 3x+1$

$du = 3.dx$

$$= \frac{1}{3} \cdot \frac{2^u}{\ln(2)} + C$$

$$= \frac{1}{3} \cdot \frac{2^{3x+1}}{\ln(2)} + C$$

$$\int \cos(x)\,dx = \sin(x) + C$$

$$\int \cos(5x)\,dx = ?$$

$$\int x \cdot \cos(x^2)\,dx$$

$$\int \cos(u)\,du = \sin(u) + C$$

case-2

cos(ugly)

u= inside

Trigonometric Functions

Example: $\int \cos(4x)\,dx = ?$

Formula: $\int \cos(u)\,du = \sin(u) + C$

$u = 4x$

$\frac{du}{dx} = 4$

$du = 4\,\boxed{dx}$

$dx = \frac{du}{4}$

$= \int \cos(u) \cdot \frac{du}{4} = \frac{1}{4} \int \cos(u) \cdot du$

formula 6.3

$= \frac{1}{4} \sin(u) + C$

$= \boxed{\frac{1}{4} \sin(4x) + C}$

ex: $\int \sin(5x)\,dx = -\frac{1}{5} \cdot \cos(5x) + C$

MUST KNOW:

$$\int \sin(mx)\,dx = \frac{-\cos(mx)}{m} + C$$

$$\int \cos(mx)\,dx = \frac{\sin(mx)}{m} + C$$

sin(ugly)

u = inside
must include details

Example: $\int x\sin\left(5x^2\right)dx = ?$

Formula: $\int \sin(u)du = -\cos(u) + C$

$u = 5x^2$

$\frac{du}{dx} = 10x$

$du = 10x\,dx$

$x\,dx = \frac{du}{10}$

$= \int \sin(u)\cdot\frac{du}{10} = \frac{1}{10}\int \sin(u)\,du$

formula 6.3

$= \frac{1}{10}\cdot -\cos(u) + C$

$= -\frac{1}{10}\cos(5x^2) + C$

Example: $\int e^x \sec^2\left(e^x\right)dx = ?$

Formula: $\int \sec^2(u)du = \tan(u) + C$

$u = e^x$

$du = e^x dx$

$= \int \sec^2(u)\,du$

formula 6.3

$= \tan(u) + C$

$= \boxed{\tan(e^x) + C}$

$f(x) = \underline{\underline{\sin}}^2(x)$

$\downarrow$

$f'(x) = \boxed{2 \cdot \sin(x) \cdot \cos(x)}$

$g(x) = \cos^3(x)$

$\downarrow$

$g'(x) = -3 \cdot \cos^2(x) \cdot \sin(x)$

product

Products of Trigonometric Functions

power rule

Example: Evaluate the integral:

$\int \sin(x)\cos(x)\,dx = \int u \cdot du = \frac{u^2}{2} + c$

$u = \sin(x)$

$du = \boxed{\cos(x) \cdot dx}$

$= \boxed{\frac{1}{2} \cdot \sin^2(x) + C}$

Example: Evaluate the integral:

$\int \cos^6(x)\sin(x)\,dx = \int u^6 \cdot -du = -\int u^6 \cdot du$

power

$u = \cos(x)$

$du = \boxed{-\sin(x)\,dx}$

$= -\frac{u^7}{7} + c$

$= -\frac{\cos^7(x)}{7} + c$

Example: Evaluate the integral:

$\int \sec^2(x)\tan(x)\,dx = \int u \cdot du = \frac{u^2}{2} + c$

easier

$u = \tan(x)$

$du = \sec^2(x)\,dx$

$= \frac{1}{2}\tan^2(x) + c$

case

$$\int x^{-1}\,dx = \int \frac{1}{x}\,dx$$

$$= \ln|x| + c$$

$$\int \frac{1}{x}\,dx$$

$$\int \frac{1}{u}\,du = \ln|u| + c$$

$$\int \frac{}{\text{ugly}}$$

$\int \frac{}{\text{ugly}}$ try u= denominator

Some rational functions (special case!):

Example: Evaluate: $\int \frac{6x}{x^2+5}dx$ $= \int \frac{3 \cdot 2x \cdot dx}{x^2+5}$

$u = x^2+5$

$\downarrow$

$du = 2x\,dx$

$= 3 \cdot \int \frac{1}{u} \cdot du$

formula 6.3

$\stackrel{\downarrow}{=} 3 \cdot \ln|u| + C$

$= \boxed{3 \cdot \ln|x^2+5| + C}$

Example: Evaluate: $\int \frac{2x+6}{x^2+6x+1}dx$

$\downarrow$

$u = x^2+6x+1$

$\downarrow$

$du = (2x+6)dx$

$= \int \frac{1}{u} \cdot du$

$= \ln|u| + C$

$= \ln|x^2+6x+1| + C$

calc 2

Example: Evaluate: $\int \frac{e^x}{5e^x+1}dx$ $= \frac{1}{5}\int \frac{1}{u}\cdot du$

$u = 5e^x+1$

$\downarrow$

$du = 5\cdot e^x\cdot dx$

$= \frac{1}{5}\ln|u| + c$

$= \boxed{\frac{1}{5}\ln|5e^x+1| + C}$

imp. for calc 2

Example: Evaluate: $\int \frac{8}{4x+1}dx$ $= 8\cdot\int \frac{1}{4x+1}\cdot dx$

$u = 4x+1$

$\downarrow$

$du = 4\cdot dx$

$= 8\cdot \int \frac{1}{u}\cdot\frac{du}{4}$

$= \frac{8}{4}\int \frac{1}{u}\cdot du$

$= 2\ln|u| + c$

$= 2\cdot\ln|4x+1| + C$

ex: 2

$$\int \frac{1}{x+1}\,dx = \int \frac{1}{u}\,du$$

$$u = x+1$$

$$du = dx$$

$$= \ln|x+1| + c$$

$$\int \frac{1}{x-2}\,dx = \ln|x-2| + c$$

Exercise:

$$\int \frac{\sin(x)}{2+\cos(x)}\,dx = ?$$

important

Rcl 2

$$\int \frac{1}{2+x} + \frac{1}{x-1} + \frac{1}{x^2}\,dx = ?$$

$$\int \frac{1}{(x-1)^2}\,dx = ? \qquad \int \frac{1}{u^2}\,du = \int u^{-2}\,du$$

$u = x-1$

$du = dx$

$u = x - 1$

$\frac{du}{dx} = 1 \Rightarrow du = dx$

The method of u-substitution with <u>Definite Integrals</u>

You either integrate and recall the limits after finding the integral in terms of x,

OR

change the limits of integration (but fund the integral only in terms of u, not x!)

Example:

OPTION 1:

$$\int_0^1 3x^2\left(x^3+2\right)^3\,dx \xrightarrow[\substack{x=0\to u=2\\ x=1\to u=3}]{u=x^3+2,du=3x^2dx} \int_2^3 u^3du = \left[\frac{u^4}{4}\right]_2^3 = \frac{3^4}{4}-\frac{2^4}{4}=\frac{65}{4}$$

OPTION 2:

$$\int_0^1 3x^2\left(x^3+2\right)^3\,dx \xrightarrow{u=x^3+2;du=3x^2dx} \int u^3du = \left[\frac{u^4}{4}+C\right] \xrightarrow{back\ to\ "x"} \left[\frac{(x^3+2)^4}{4}+C\right]_0^1$$

$$=\frac{3^4}{4}-\frac{2^4}{4}=\frac{65}{4}$$

$\cancel{\int_0^2 \frac{4x}{x^2+1}\,dx = \int_0^2 \frac{2}{u}\,dx}$

Final

Example: Evaluate:

$$\int_0^2 \frac{\overbrace{4x}^{2.2x\,dx}}{x^2+1}\,dx = \int \frac{2}{u}.du = 2\ln|u|+c = \left[2\ln|x^2+1|+c\right]_0^2$$

$$= 2.\ln(5) - 2\ln(1)$$

$$= \boxed{2\ln(5)}$$

$\boxed{u = x^2+1}$

$du = 2x.dx$

$x=0 \Rightarrow u = 0^2+1 = 1$

$x=2 \Rightarrow u = 2^2+1 = 5$

2nd

$$\int_1^5 \frac{2}{u}du = \left[2\ln|u|\right]_1^5 = 2\ln(5) - 2\ln(1)$$

$$= \boxed{2\ln(5)}$$

Example: Evaluate:

$$\int_0^{\pi/2} \frac{\cos x}{2+\sin x}\,dx = \int_2^3 \frac{1}{u}\,du = \left[\ln|u|\right]_2^3 = \boxed{\ln(3) - \ln(2)}$$

$u = 2+\sin(x)$

$du = \cos(x)dx$

$x = 0 \Rightarrow u = 2+\sin(0) = 2+0 = 2$

$x = \pi/2 \Rightarrow u = 2+\sin(\frac{\pi}{2}) = 2+1 = 3$

Here is a summary of the formulas we covered in this section:

TABLE OF INTEGRALS

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + C, \text{ for } n \neq -1.$$

$$\int \frac{1}{u} \, du = ln|u| + C \qquad \text{and} \qquad \int \frac{g'(x)}{g(x)} dx = ln|g(x)| + C$$

$$\int tan\, x \, dx = -ln|cos\, x| + C = ln|sec\, x| + C$$

$$\int sec\, x \, dx = ln|sec\, x + tan\, x| + C$$

Calc 2

on final!

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = arcsin\left(\frac{x}{a}\right) + C \qquad \int \frac{1}{\sqrt{a^2 - u^2}} \, du = arcsin\left(\frac{u}{a}\right) + C$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} arctan\left(\frac{x}{a}\right) + C \qquad \int \frac{1}{a^2 + u^2} \, du = \frac{1}{a} arctan\left(\frac{u}{a}\right) + C$$

Homework: Read Section 6.4 from your text book to see how these formulas are derived. Study the examples there.

$$\int \frac{1}{1+x^2}\,dx = \arctan(x)$$

Study these solved examples (applications of formulas). These type of integrals will show up a lot in Calculus 2.

FORMULA: $\boxed{\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right)+C}$

reason
u-sub

Ex: $\int \frac{1}{4+x^2}\,dx = \frac{1}{2}\arctan\left(\frac{x}{2}\right)+C$

Ex: $\int \frac{2}{9+x^2}\,dx = 2\int \frac{1}{9+x^2}\,dx = \frac{2}{3}\arctan\left(\frac{x}{3}\right)+C$

Ex:

$$\int \frac{2}{9+25x^2}\,dx = 2\int \frac{1}{9+(5x)^2}\,dx \xrightarrow[du=5dx]{u=5x} 2\int \frac{1}{9+u^2}\cdot\frac{du}{5} = \frac{2}{5}\cdot\frac{1}{3}\arctan\left(\frac{u}{3}\right)+C$$

$$= \frac{2}{15}\arctan\left(\frac{5x}{3}\right)+C$$

$\text{number}^2 + \boxed{x^2}$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x) + C$$

FORMULA: $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right) + C$

Ex: $\int \frac{1}{\sqrt{4-x^2}}\,dx = \arcsin\left(\frac{x}{2}\right) + C$

Ex: $\int \frac{2}{\sqrt{9-x^2}}\,dx = 2\int \frac{1}{\sqrt{9-x^2}}\,dx = 2\arcsin\left(\frac{x}{3}\right) + C$

Ex:

$$\int \frac{2}{\sqrt{9-49x^2}}\,dx = \int \frac{2}{\sqrt{9-(7x)^2}}\,dx \xrightarrow[du=7dx]{u=7x} \frac{2}{7}\int \frac{1}{\sqrt{9-u^2}}\,du = \frac{2}{7}\arcsin\left(\frac{u}{3}\right) + C$$

$$= \frac{2}{7}\arcsin\left(\frac{7x}{3}\right) + C$$

calc 2

NOTE: The integrals $\int \frac{x}{\sqrt{4-x^2}}\,dx$ or $\int \frac{x}{4+x^2}\,dx$ are NOT about these formulas: they are typical "u-sub" questions. Do you see why?

$$\int \frac{\boxed{x}}{4+x^2}\,dx \quad \text{vs} \quad \int \frac{5}{4+x^2}\,dx$$

$$5 \cdot \int \frac{1}{4+x^2}\,dx$$

Remark: Make sure you know how to integrate the following: (Very important!)

$$\int \frac{ax}{x^2+c}dx$$

u-sub

u= denom.

$$\int \frac{b}{x^2+c}dx$$

arctan

$$\int \frac{ax+b}{x^2+c}dx = \underbrace{\int \frac{ax}{x^2+c}dx}_{\text{u-sub}} + \underbrace{\int \frac{b}{x^2+c}dx}_{\text{arctan}}$$

see next pages !!!

MISCALLENAUS EXERCISES:

Compute the following integrals:

exc: $\int_0^2 \frac{2x}{x^2+5}dx = ?$

important for final

* 1) $\int \frac{2x}{x^2+5}dx = \int \frac{1}{u}du$

$u = x^2+5$

2) $\int \frac{4x+1}{x^2+1}dx$

3) $\int \frac{4}{x+1}+\frac{2}{x-1}dx$

4) $\int \frac{4x+8}{\sqrt{\underbrace{x^2+4x+1}}}dx = 2\int \frac{1}{\sqrt{u}}du$

$u = x^2+4x+1$

5) $\int_0^1 \frac{6x+12}{\sqrt{x^2+4x+1}}dx =$

* 6) $\int_0^1 x\left(x^2+1\right)^3 dx = \frac{1}{2}\int_1^2 u^3 du = \frac{1}{2}\cdot\frac{u^4}{4}\Big]_1^2 = \frac{1}{8}\left(\frac{16}{4}-\frac{1}{4}\right)$

$u = x^2+1$

$= 15/32.$

7) $\int \frac{e^x}{e^{2x}+4}dx$

8) $\int \frac{e^x}{e^x+4}dx$

9) $\int \frac{4x+8}{x^2+4x+1}dx = 2\int \frac{1}{u}du = 2\ln|x^2+4x+1|+C$

$u = x^2+4x+1$

$du = 2x+4$

10) $\displaystyle\int_{0}^{\pi} \sin(x)\sqrt{\underbrace{8+\cos(x)}_{u=8+\cos(x)}}\,dx =$ $\rightarrow$ $\displaystyle\int_{9}^{7} \sqrt{u}\,du$

11) $\displaystyle\int_{0}^{\pi/2} \sin(x)\cos^2(x)\,dx =$

12) $\displaystyle\int \sin(2x)+\cos(4x)\,dx =$ $-\frac{1}{2}\cos(2x)+\frac{1}{4}\sin(4x)+C$

* 13) $\displaystyle\int \left(e^{5x}+\frac{1}{x^2}\right)dx =$ $\frac{1}{5}e^{5x}-\frac{1}{x}+C$

14) $\displaystyle\int \frac{1}{(x+1)^2}\,dx = ?$

15) $\displaystyle\int \frac{1}{(2x+1)^2}\,dx = ?$

$$* \quad \int_0^1 \frac{2x}{x^2+5}\,dx = \int_5^6 \frac{1}{u}\,du$$

$u = x^2+5$

$du = 2x\,dx$

$x=0 \Rightarrow u=5$

$x=1 \Rightarrow u=6$

$$= \Big[\ln|u|\Big]_5^6$$

$$= \boxed{\ln(6) - \ln(5)}$$

final answer

exc

$$* \quad \int_0^2 \frac{6x}{3x^2+2}\,dx = \int_2^{14} \frac{1}{u}\,du$$

$u = 3x^2+2$

$du = 6x\,dx$

$x=0 \Rightarrow u=2$

$x=2 \Rightarrow u=14$

$$= \Big[\ln|u|\Big]_2^{14}$$

$$= \boxed{\ln(14) - \ln(2)}$$

example (important):

$$\int_0^2 \frac{10x}{5x^2+9}\,dx = \int_9^{29} \frac{1}{u}\,du$$

formula

$u = 5x^2+9$

$du = 10x\,dx$

$x=0 \Rightarrow u = 9$

$x=2 \Rightarrow u = 5\cdot 4+9 = 29$

$$= \Big[\ln|u|\Big]_9^{29}$$

plug in u-bound.

$$= \underbrace{\ln(29) - \ln(9)}$$

final answer