Practice Final

MAT1451 Sec: 18804 Spring 2018

Name:(Print)________________________________

ID number:________________________________

Do not turn the page until instructed to do so.

No books, notes, calculators, laptops, cell phones or any other help is permitted on the test.

Show all work, points will be deducted if work is sloppy or not shown. Write your arguments in a logical, well-organized and clear way.

If you need more paper use the back sides of the pages or ask for additional scratch paper. Do not forget to write your name, ID and problem number on the scratch paper and attach it to your exam.

The duration of the exam is 50 min.

You are expected to adhere to the Academic UH Honesty Policy.

Take a deep breath, and good luck!

Do not write anything on this page!

Problem	Score
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total points	
Percentage	

1. (a) Find the Maclaurin series for the function

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

   (b) Use the series to evaluate the limit

$$\lim_{x\to 0} \frac{\ln(1-x^2)}{1-\cos x}$$

2. (a) For which values of a will the vectors $\boldsymbol{u} = 2\boldsymbol{i} + 4\boldsymbol{j} - 5\boldsymbol{k}$ and $\boldsymbol{v} = -4\boldsymbol{i} - 8\boldsymbol{j} + a\boldsymbol{k}$ be parallel?

   (b) Suppose that $\boldsymbol{n}$ is normal to a plane, and that $\boldsymbol{v}$ is parallel to the plane. Find a vector that is both perpendicular to $\boldsymbol{v}$ and parallel to the plane.

   (c) Find a unit vector orthogonal to $\boldsymbol{A} = 2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ in the plane of both $\boldsymbol{B} = \boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{C} = \boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$.

3. (a) For the curve

$$\boldsymbol{r}(t) = (e^t \sin 2t)\boldsymbol{i} + (e^t \cos 2t)\boldsymbol{j} + 2e^t\boldsymbol{k}$$

   at $t = 0$ find

   i. $\boldsymbol{T}$

   ii. $\boldsymbol{N}$

   iii. $\boldsymbol{B}$

   iv. and curvature κ

   (b) Find parametric equation for the line that is tangent to the curve of intersection of surfaces $x^2 + 2y + 2z = 4$ and $y = 1$ at point $(1, 1, 1/2)$.

4. For the function

$$f(x, y) = \ln(2x + 3y + 6z)$$

and the point $P_0(-1, -1, 1)$

   (a) find the directions in which f increases most rapidly at P_0

   (b) find the directions in which f decreases most rapidly at P_0

   (c) find the derivative of f at the direction of $\boldsymbol{v} = 2\boldsymbol{i} + 3\boldsymbol{j} + 6\boldsymbol{k}$ at P_0

5. (a) Find the absolute maximum and minimum of $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2$ on the square region enclosed by the lines $x = \pm 1$ and $y = \pm 1$.

   (b) Find extreme values of $f(x, y, z) = x - y + z$ on the unit sphere $x^2 + y^2 + z^2 = 1$.

6. (a) Find the area of the region enclosed by the line $y = 2x + 4$ and the parabola $y = 4 - x^2$ in xy-plane.

   (b) Find the volume of the region bounded above by the sphere $x^2 + y^2 + z^2 = 8$ and below by the plane $z = 2$

i. by using cylindrical coordinates

ii. by using spherical coordinates

7. Evaluate
$$\int\int\int |xyz| dx\, dy\, dz$$
over the solid ellipsoid
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1$$

8. Integrate $\boldsymbol{F} = -(y \sin z)\boldsymbol{i} + (x \sin z)\boldsymbol{j} + (xy \cos z)\boldsymbol{k}$ around the circle cut from the sphere $x^2+y^2+z^2 = 5$ by the plane $z = -1$, clockwise as viewed from above.

9. Evaluate the integral
$$\oint_C (y^2 dx + x^2 dy)$$
where C is the triangle bounded by $x = 0$, $a + y = 1$ and $y = 0$

   (a) directly

   (b) applying Green's Theorem

10. (a) Find the area of the cap cut from the paraboloid $y^2 + z^2 = 3x$ by the plane $x = 1$.

    (b) Verify that Stoke's Theorem is true for the vector field $\boldsymbol{F}(x, yz) = x^2\boldsymbol{i} + y^2\boldsymbol{j} + z^2\boldsymbol{k}$, where S is the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the xy-plane and S has upward orientation.