Math 1314
Final Exam Review

- The Final Exam covers all course material from Lessons 1 – 24.
- Do Practice Final Exam for extra practice and extra credit.
- Review all other test reviews given during the semester.
- Do the teacher evaluations in CASA for an extra 5 points extra credit on the Final Exam.
- If the Final Exam raw score is better than your lowest regular raw test grade, it will replace it.
- The downloaded version of GGB (not the web version) will be a link on the CASA Testing computers.
- The following formulas will be provided on the Final Exam CASA Testing Center computers as a link.

\[
\frac{f(x+h) - f(x)}{h} = \frac{f(b) - f(a)}{b-a}
\]

\[
C(x) = cx + F
\]
\[
R(x) = sx \quad \text{or} \quad R(x) = xp
\]
\[
P(x) = R(x) - C(x)
\]
\[
\overline{C}(x) = C(x)
\]
\[
E(p) = -\frac{p \cdot f'(p)}{f(p)}
\]
\[
CS = \int_0^{Q_E} D(x) \, dx - Q_E \cdot P_E
\]
\[
PS = Q_E \cdot P_E - \int_0^{Q_E} S(x) \, dx
\]
\[
\frac{1}{b-a} \int_a^b f(x) \, dx
\]
\[
D(x, y) = f_{xx} f_{yy} - (f_{xy})^2
\]
- If \( D(a, b) > 0 \) and \( f_{xx}(a,b) < 0 \), then \( f \) has a relative maximum at \((a, b)\).
- If \( D(a, b) > 0 \) and \( f_{xx}(a,b) > 0 \), then \( f \) has a relative minimum at \((a, b)\).
- If \( D(a, b) < 0 \), then \( f \) has neither a relative maximum nor a relative minimum at \((a, b)\) (i.e., it has a saddle point, which is neither a max nor a min).
- If \( D(a, b) = 0 \), then this test is inconclusive.
1) The following table of values gives a company’s annual profits in millions of dollars. Rescale the data so that the year 2001 corresponds to $x = 0$.

<table>
<thead>
<tr>
<th>Year</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profits (in millions of dollars)</td>
<td>23.8</td>
<td>25.2</td>
<td>26.3</td>
<td>28.9</td>
<td>27.6</td>
<td>28.4</td>
</tr>
</tbody>
</table>

Find the quadratic regression model for this data.

a) $y = -0.3214x^2 + 2.6986x + 22.72$

b) $y = -0.2786x^2 + 2.2443x + 23.5429$

c) $y = -0.2419x^2 + 2.2278x + 23.5057$

d) $y = -0.0964x^2 + 1.3679x + 23.8643$

e) $y = -0.225x^2 + 2.0621x + 23.6071$

f) None of the above

2) Given the following graph of a function $f$. Determine $\lim_{x \to 2} f(x)$, if possible.
3) Evaluate each limit below, if possible.

a. \( \lim_{{x \to -1}} \sqrt{-x^2} + 5 \)

b. \( \lim_{{x \to 7}} \frac{x^2 + 49}{x-7} \)

c. \( \lim_{{x \to 1}} \frac{x^2 - 1}{x - 1} \)

d. \( \lim_{{x \to \infty}} \frac{2x^2 - x}{10x^2 - 1} \)
4) Given the graph below of a function $f$, name the type of discontinuity at:
   a. $x = -2$  
   b. $x = -0.5$  
   c. $x = 0$  
   d. $x = 3$

[Graph of a function $f(x)$ showing points at $x = -2$, $x = -0.5$, $x = 0$, and $x = 3$]

5) Give the second derivative of $f(x) = \frac{1}{3}x^3 - 4x^2 - x + 1$.

6) Let $f(x) = x^2 + 4x - 5$.
   a. Give the slope of the tangent line when $x = 1$.

   b. Find any value(s) of $x$ where the tangent line is horizontal to the function $f$. 

7) A person standing on top of a building that is 112 feet high. This person throws a rock vertically upward with an initial velocity of 84 feet per second. The equation that represents this situation is \( h(t) = -16t^2 + 84t + 112 \). What is the velocity of the rock at 2 seconds?

8) At the beginning of an experiment, a researcher has 562 grams of a substance. If the half-life of the substance is 26 days, how many grams of the substance are left after 45 days?

9) A company’s fixed costs are $1,500. It costs the company $100 to produce an item and the item sells for $125. Give the profit function and find the breakeven quantity. *Assume the functions are linear and all produced items are be sold.*
10) A computer company manufactures a certain variety of flat panel monitor. The demand for this monitor is given by the following equation, where $p$ denotes the unit price and $x$ denotes the quantity demanded.

$$p = -0.08x + 400 \text{, where } 0 \leq x \leq 5000$$

Use the marginal revenue function to approximate the actual revenue realized on the sale of the 500th monitor.

11) The graph of the first derivative of a polynomial function $f$ is given below. Find any critical numbers and the x-coordinate(s) of any relative extrema of $f$. 

![Graph of the first derivative of a polynomial function](image_url)
12) The graph of the second derivative of a polynomial function $f$ is given below. Find the $x$-coordinate(s) of any points of inflection of $f$.

![Graph of $f''(x)$](image)

13) Suppose $f(x) = \frac{x^3 - 4x^2 - 8}{x}$. Find any intervals of increase/decrease and any relative extrema.

*Enter the function into GGB, the function is graphed below.*

![Graph of $f(x)$](image)
14) Given the function

\[ f(x) = \frac{1}{6}x^4 - x^3 + 2x^2 - x + 1 \]

Find the interval(s) on which the graph of the function is concave up and the interval(s) on which the graph of the function is concave down.

a) Concave up on \((-\infty, 1]\); concave down on \([1, \infty)\).
b) Concave up on \((-\infty, 1) \cup (2, \infty)\); concave down on \((1, 2)\).
c) Concave up on \((-\infty, -2]\); concave down on \((-2, \infty)\).
d) Concave up on \((-\infty, 2]\); concave down on \((2, \infty)\).
e) Concave up on \((-\infty, 2]\); concave down on \((-\infty, 1) \cup (2, \infty)\).
f) None of the above

**Enter the function into GGB, the function is graphed below.**

![Graph of the function](image)

15) Suppose \(f(x) = \frac{1}{2}xe^{-5x^2}\). Find any intervals of concavity.

a) Concave upward on \((-\infty, 0)\); concave downward on \((0, \infty)\)
b) Concave upward on \((-0.3162, 0.3162)\); concave downward on \((-\infty, -0.3162) \cup (0.3162, \infty)\)
c) Concave upward on \((0, \infty)\); concave downward on \((-\infty, 0)\)
d) Concave upward on \((-0.5477, 0) \cup (0.5477, \infty)\); concave downward on \((-\infty, -0.5477) \cup (0, 0.5477)\)
e) Concave upward on \((-\infty, -0.5477) \cup (0, 0.5477)\); concave downward on \((-0.5477, 0) \cup (0.5477, \infty)\)
f) None of the above

**Enter the function into GGB, the function's second derivative is graphed below.**

![Graph of the function's second derivative](image)
16) Find the coordinates of any absolute minimum of $f(x) = 3x^3 - 9x^2 + 5$ on the interval $[-1, 3]$.

17) A storeowner wants to set up a rectangular display area outside his store. He will use garage (which is 35 feet long) as part of one side of the display area. He has 230 linear feet of fencing material to use to fence in the display area. Write a function which expresses the total area of the display area in terms of the length $x$ of the side of the display area opposite the building.

Then find the maximum area.
18) Let \( f(x) = 2x^2 + 1 \). Compute the Riemann sum of \( f \) over the interval \([0, 4]\) using 4 subintervals, choosing the midpoints of the subintervals as representative points.

a) \( \bigcirc 65 \)

b) \( \bigcirc 46 \)

c) \( \bigcirc 25 \)

d) \( \bigcirc 32 \)

e) \( \bigcirc 64 \)

f) \( \bigcirc \) None of the above

19) Find the indefinite integral \( \int (3x^3 + 5x - 1) \, dx \).

20) Suppose the velocity of a car can be modeled by the function

\[
v(t) = 6t \sqrt{16 - t^2}
\]

where \( t \) represents the time in seconds and \( v(t) \) is given in feet per second. Find the total distance traveled by the car from \( t = 0 \) to \( t = 3 \).

a) \( \bigcirc 378.00 \) feet

b) \( \bigcirc 28.54 \) feet

c) \( \bigcirc 26.22 \) feet

d) \( \bigcirc 90.96 \) feet

e) \( \bigcirc 13.19 \) feet

f) \( \bigcirc \) None of the above
21) The temperature in Minneapolis over a 12 hour period can be modeled by the function

\[ C(t) = -0.06t^3 + 0.6t^2 + 4.1t + 5.3 \]

where \( t \) is measured in hours with \( t = 0 \) corresponding to the temperature at 12 noon. Find the average temperature during the period from noon until 7 p.m.

a) \( \bigcirc \) 223.2 degrees

b) \( \bigcirc \) 24.3 degrees

c) \( \bigcirc \) 44.6 degrees

d) \( \bigcirc \) 0 degrees

e) \( \bigcirc \) 170.1 degrees

f) \( \bigcirc \) None of the above

22) Let \( f(x) = x^2 - 6x \) and \( g(x) = 2x \). Set up an integral to find the area of the region between the two functions, then find the exact area.
23) Suppose the demand function for a product is \( x \) thousand units per week and the corresponding wholesale price, in dollars, is

\[ p = \sqrt{-9x + 174} \]

Determine the consumers' surplus if the wholesale market price is set at $8 per unit.

a) $38,204  
b) $229,512  
c) $59,396  
d) $34,312  
e) $131,912  
f) None of the above

24) Find any critical points of \( f(x, y) = 4 + x^3 + y^3 - 3xy \). Then use the second derivative test to classify each critical points as a relative maximum, relative minimum or saddle point.