Let’s revisit the different ways we can solve quadratic equations.

- Solving by Factoring.
- Square root method.
- Completing the Square.
  1. Rewrite the equation as \( x^2 + bx = -c \) (Notice that the leading coefficient is positive 1, if it’s not then you will have to divide both sides of the equation by the leading coefficient.) and make the left-hand side a perfect square.
  2. Make the left-hand side a perfect square by adding \( \left( \frac{b}{2} \right)^2 \) to both sides (to balance the equation)
  3. Factor the left-hand side.
  4. Use the square root property to solve.

- Quadratic Formula

Given \( ax^2 + bx + c = 0 \), then \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \).

Other times we may want to simply want to know how many solutions a quadratic equation has. We can use the discriminant of the equation to find just that.

\[ D = b^2 - 4ac. \]

- If \( D > 0 \), then the equation \( ax^2 + bx + c = 0 \) has two distinct real solutions.
- If \( D = 0 \), then the equation \( ax^2 + bx + c = 0 \) has exactly one real solution.
- If \( D < 0 \), then the equation \( ax^2 + bx + c = 0 \) has no real solution (The roots of the equation are complex numbers and appear as complex conjugate pairs.)

Example 1: How many real solutions does \( 3x^2 + 2x + 2 = 0 \) contain?
Example 2: Solve each equation.

a. \( \frac{2x}{x-3} + \frac{6}{x+3} = \frac{28}{x^2-9} \)
b. $\frac{1}{x} + \frac{1}{x+3} = \frac{1}{4}$
c. $2x^2 - 4x - 1 = 0$
d. A park is rectangular in shape and has a length that is 3 yards greater than the width. The area of the park is 180 square yards. Find the length.