Section 2.4
An Introduction to Complex Numbers

A complex number is a number that can be written in the form \( a + bi \), where \( a \) is called the real part and \( bi \) is called the imaginary part. The \( a \) and \( b \) are real numbers.

The imaginary unit \( i \) is defined as \( i = \sqrt{-1} \), where \( i^2 = -1 \).

Complex numbers are used in many fields in real life. For example in Electronics: Circuit is mainly based on current and voltage. Those two elements are put together as a single complex number: \( Z = V + iI \) This complex representation shows a circuit having both current and voltage.

Addition or Subtraction of Complex Numbers:
Add or subtract the real parts together and add the imaginary parts together.

Multiplication of Complex Numbers:
Multiply in the same manner as multiplying binomials and remember that \( i^2 = -1 \)

Example 1: Simplify each of the following and write the answer in form \( a + bi \).
\[-3i(2 + i) + (-1 + 2i)^2\]
Example 2: Simplify each expression to $a + bi$. Recall: $i = \sqrt{-1}$

a. $\sqrt{-27} \cdot \sqrt{-4} - \sqrt{-16} + \sqrt{3}$

b. $\frac{\sqrt{-8}}{\sqrt{-36} \cdot \sqrt{-100}}$
Powers of $i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

In fact, if $k$ is a multiple of 4 then $i^k = 1$.

Example 3: Simplify $i^{18}$.
**Division of Complex Numbers**

The **complex conjugate** of the complex number \( a + bi \) is the complex number \( a - bi \).

To simplify the quotient \( \frac{a + bi}{c + di} \), multiply both the numerator and denominator by the complex conjugate of the denominator.

Example 4: Simplify the following expression and write the answer in form \( a + bi \).
\[
\frac{3 - 4i}{1 - 3i}
\]
Complex Roots of Quadratic Equations

Example 5: Find all complex solutions of the following equation. Express your answer in form $a + bi$.

a. $-x^3 - x = 0$

b. $2x^2 + 20x = -84$