Section 2.2
Derivatives of Polynomials and Trigonometric Functions

As you saw in several examples in Section 2.1, finding the derivative of a function using the definition is quite tedious. We now introduce some rules that will make our computations much easier.

A constant function is represented by a horizontal line. What is the slope of a horizontal line?

Then the derivative of any constant is zero!

**Rule 1: The Derivative of a Constant**

$$\frac{d}{dx}[k] = 0, \text{ where } k \text{ is a constant.}$$

Example 1: Find the derivative of $f(x) = -17$.

**Rule 2: The Power Rule**

$$\frac{d}{dx}[x^n] = nx^{n-1} \text{ for any real number } n$$

Example 2: Find the derivative of each function.

a. $f(x) = x^5$

b. $g(x) = x^{-10}$
c. \( j(x) = \sqrt{x} \)

**Rule 3: Derivative of a Constant Multiple of a Function**

\[
\frac{d}{dx} [kf(x)] = k \frac{d}{dx} [f(x)] \quad \text{where } k \text{ is any real number}
\]

Example 3: Find the derivative of each function.

a. \( f(x) = -6x^4 \)

b. \( g(x) = \frac{1}{4x^4} \)
Theorem: Let $k$ be any real number. If $f$ and $g$ are differentiable at $x$, then so are $f + g$ and $f - g$. Moreover, \( (f \pm g)'(x) = f'(x) \pm g'(x) \).

Example 4: Find the derivative: \( f(x) = \frac{3}{8} x^6 + \frac{3}{x^2} + 9\sqrt[3]{x} - 5 \)

Example 5: Find the derivative: \( f(x) = \frac{x^5 + 10x^3 + 1}{x^2} \)
Derivatives of the Trigonometric Functions

\[
\begin{align*}
\sin x)' &= \cos x \\
\cos x)' &= -\sin x \\
\tan x)' &= \sec^2 x \\
\cot x)' &= -\csc^2 x \\
\sec x)' &= \sec x \tan x \\
\csc x)' &= -\csc x \cot x
\end{align*}
\]

Example 6: Find the slope of the tangent line to the function \( f(x) = 6 \tan x - \cos x \) at \( x = -\pi \).

In other cases, we may want to find all points for which the tangent line to the graph of \( f \) is horizontal or equal to a specified number. Set the derivative equal to the given number and solve for \( x \).

Example 7: Find all values of \( x \) in \([0, 2\pi]\) where the tangent line is horizontal to \( f(x) = 10\sqrt{3} \sin x - 10 \cos x \).
Example 8: Find all x-value(s) on the graph of \( f(x) = x^3 + 5x^2 \) where the slope of the tangent line is equal to -3.

**Tangent and Normal Lines**

We already known what a tangent line is all about.

A **normal line** to a curve at a particular point is the line through that point and perpendicular to the tangent.

Recall from Pre-Algebra the slope of any line perpendicular to a line with slope is the negative reciprocal.

Example 9: Find the equations of the tangent and normal line to \( f(x) = x^3 + 2x \) at \( x = 1 \).
Higher Order Derivatives

\[ f'(x), \quad f''(x), \quad f'''(x), \quad f^{(4)}(x) \]

\[ \frac{d}{dx} f(x), \quad \frac{d^2}{dx^2} f(x), \quad \frac{d^3}{dx^3} f(x), \quad \frac{d^4}{dx^4} f(x), \]

Example 10: Find \( f''(x) \) for \( f(x) = 6 \sin x \).

Now that we know much easier rules for finding derivatives, let’s revisit differentiability with piecewise functions.

Example 11: Is this function differentiable?

\[ f(x) = \begin{cases} 
2x, & \text{if } x > 1 \\
x^3 + 1, & \text{if } x \leq 1 
\end{cases} \]

Continuity must first be checked…after checking with the 3-step method (see Section 1.4), the function is continuous at \( x = 1 \).
Example 12: Given \( f(x) = \begin{cases} Bx + C, & \text{if } x > 1 \\ 8 + x^3, & \text{if } x \leq 1 \end{cases} \). Determine the values of the constants \( B \) and \( C \) so that \( f \) is differentiable.
Try this one:

Let $f(x) = \tan x - x$. Find all values of $x$ in $[0, 2\pi]$ where the tangent line is horizontal.