The geometric distribution is the distribution produced by the random variable $X$ defined to count the number of trials needed to obtain the first success.

**Examples of a Geometric experiment:**
- Flipping a coin until you get a head.
- Rolling a die until you get a 5.

A random variable $X$ is geometric if the following conditions are met:

1. The geometric random variable $X$ is defined to be the number of trials until the first success is observed.
2. Each trial can result in one of only two mutually exclusive outcomes (success or failure).
3. Outcomes of different trials are independent.
4. The probability that a trial results in success is the same for all trials.

Notice that this is different from the binomial distribution in that the number of trials is unknown. With geometric distributions we are trying to determine how many trials are needed in order to obtain a success.

Example 1: Determine whether the following problem is a geometric experiment.

a. In football, attempt a field goal from the 50 yard line until you make one.

b. You make a $5 bet on the number 00 in roulette until you win.

c. In basketball, you give yourself two chances to make a three-point shot.
In a **geometric experiment**, to find the probability that the first success occurs on the \( n \)\(^{\text{th}} \) trial is given by: \( P(X = n) = (1 - p)^{n-1} \cdot p \), where \( p \) is the probability of success.

R command: \( P(X = n) = \text{dgeom}(n - 1, p) \)

Example 2: Sally is outside a polling station. She asks people there who they voted for until she finds someone that voted for the independent candidate in the last election. If the probability of success is 0.18, what is the probability that she meets an independent voter on her third try?

Command: 

Answer:

In a **geometric experiment**, to find the probability that it takes \( n \) or fewer trials to observe the first success then:

R command:

\( P(X \leq n) = \text{pgeom}(n - 1, p) \)

Example 3: There is a probability of 0.08 that a vaccine will cause a certain side effect. Suppose that a number of patients are inoculated with the vaccine. Find the probability that

a. 7 or fewer patients must be vaccinated in order to observe the first side effect.

Command: 

Answer:

b. more than 5 patients must be vaccinated in order to observe the first side effect.

Command: 

Answer:
Geometric Distribution Mean, Variance and Standard Deviation Formulas

Mean: \( E(X) = \mu = \frac{1}{p} \).

Variance: \( \sigma^2 = \frac{1-p}{p^2} \).

Standard Deviation: \( \sigma = \sqrt{\frac{1-p}{p^2}} \).

Example 4: Newsweek in 1989 reported that 60% of young children have blood lead levels that could impair their neurological development. Assuming a random sample from the population of all school children at risk, how many students would you expect to have tested in order to find the first one to have these blood levels?

Example 5: Find the probability a die must be rolled more than 4 times before a three comes up.

Command: Answer: