Section 4.2
The Normal Distribution

A density curve that is symmetric, single peaked and bell shaped is called a normal distribution. The mean, median and the mode are all equal in a normal distribution and they occur at the peak. 50% of the values are less than the mean, and 50% are greater than the mean. Probabilities are represented by areas under the curve.

Many real world things closely follow a Normal Distribution such as:
- Heights of men/women/children
- Blood pressure
- Exam scores

When the standard deviation is calculated, generally it is found that:

1. Approximately 68% of all observations fall within one standard deviation of the mean.
2. Approximately 95% of all observations fall within two standard deviations of the mean.
3. Approximately 99.7% of all observations fall within three standard deviations of the mean.

This is called the Empirical Rule.

Example 1: Assuming normality, if 95% of students at a certain school are between 1.1 m and 1.7 m tall, calculate the mean and the standard deviation then sketch the graph of its distribution.
Example 2: The length of time needed to complete a certain test is normally distributed with mean 60 minutes and standard deviation 10 minutes. First sketch the distribution.

- a. What is the probability that someone will take between 40 and 80 minutes to complete the test?

- b. What percent of people take more than 80 minutes to complete the test?

- c. What percent of people take between 70 and 80 minutes to complete the test?

- d. Find the interval that contains the middle 99.7% of completion times for all people taking the test.
What if we wanted to solve problems where we could not strictly use the Empirical Rule? RStudio has a nice command that'll calculate this for us!

Using R, the probability $P(X < x)$ can be found with the command `pnorm(x, \mu, \sigma)`.

*Note: The command in R only gives the probability that $X$ is less than a given value. If we need to find the probability that $X$ is greater than the given value, we will need to subtract the answer from 1. See the pictures below.*

R will only give us area to the left of a value, i.e. $P(X < x)$. Command: `pnorm(x, \mu, \sigma)`

![Diagram of normal distribution with area to the left shaded]

To calculate area to the right of a number, $P(X > x)$, command: $1 - \text{pnorm}(x, \mu, \sigma)$

![Diagram of normal distribution with area to the right shaded]

To calculate area between two numbers, $P(a < X < b)$, command: `pnorm(b, \mu, \sigma) - \text{pnorm}(a, \mu, \sigma)`

![Diagram of normal distribution with area between two numbers shaded]
Example 3: The average wind speed in a certain northern city in August is 13.6 miles per hour and the standard deviation is 6 miles per hour. Suppose the distribution is normal.

a. What is the probability that a randomly chosen day in August the wind speeds are less than 7.2 miles per hour?

Command: Answer:

b. What is the probability that a randomly chosen day in August the wind speeds are greater than or equal to 25 miles per hour?

Command: Answer:

c. What is the probability that a randomly chosen day in August the wind speeds are between 14 and 22 miles per hour?

Command: Answer:
Now let’s say that we know a probability and want to find the $x$ value such that:

- $P(X < c) = p$, command: `qnorm(p, μ, σ)`
- $P(X > c) = p$, command: `qnorm(1 - p, μ, σ)`
- $P(-c < X < c) = p$, command: `qnorm((p+1)/2, μ, σ)`

Example 4: Assuming normality for a random variable $X$ with mean 70 and standard deviation 10 find the value of $X$ larger than 92% of all $X$-values.

Command: 

Answer:

Example 5: Let’s revisit Example 2: “The length of time needed to complete a certain test is normally distributed with mean 60 minutes and standard deviation 10 minutes.” How long would it take someone to finish the test if they are in the top 3% of the times?

Command: 

Answer:

Example 6: The heights of a group of women aged 18 – 24 are approximately normally distributed with a mean of 65 inches and standard deviation 1.6 inches. About 25% of women have heights above what height?

Command: 

Answer: