Office Hours: Tuesdays & Thursdays 11:45-1:15
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PT 1 Questions
Section 1.2 - The Idea of a Limit
Section 1.2 - The Idea of a Limit

What is $f(2.999999)$ really close to?
What is $f(3.0000001)$ really close to?
Section 1.2 - The Idea of a Limit

What do we “expect” $f(3)$ to be based on your answers to the previous two slides?
Section 1.2 - The Idea of a Limit

This expected value of $f(3)$ is the limit of this function as $x$ approaches 3.

![Graph of a function $f(x)$ with points at (1,1), (2,2), (3,3), and a limit approaching as $x$ approaches 3.]
Section 1.2 - The Idea of a Limit

Notation: \( \lim_{x \to 3} f(x) = 2 \)
Section 1.2 - The Idea of a Limit
Section 1.2 - The Idea of a Limit

What is $f(3)$?
What is $\lim_{x \to 3} f(x)$?
Section 1.2 - The Idea of a Limit

![Graph showing limits](image-url)
Section 1.2 - The Idea of a Limit

Notation:

The limit as $x$ approaches $a$ from the left: $\lim_{x \to a^-} f(x)$

The limit as $x$ approaches $a$ from the right: $\lim_{x \to a^+} f(x)$

Rule:

For a limit to exist at $x = a$, we need $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$
Section 1.2 - The Idea of a Limit

Below is the graph of $f(x) = \sin\left(\frac{1}{x}\right)$. What is $\lim_{x \to 0} f(x)$?
Section 1.2 - The Idea of a Limit

Let’s look at \( f(x) = \frac{|x|}{x} \).

To graph this, it helps to know what \(|x|\) means:

So, \( \frac{|x|}{x} = \)
Section 1.2 - The Idea of a Limit

\[ f(x) = \frac{|x|}{x} : \]
Section 1.2 - The Idea of a Limit

Given \( f(x) = \frac{|x|}{x} \), find the following limits:

1. \( \lim_{x \to 2} f(x) \)
2. \( \lim_{x \to -3} f(x) \)
3. \( \lim_{x \to 0^+} f(x) \)
4. \( \lim_{x \to 0^-} f(x) \)
5. \( \lim_{x \to 0} f(x) \)
Limits: The Main Idea

The limit of \( f(x) \) as \( x \) approaches the value “\( a \)” gives the behavior of \( f(x) \) near \( x = a \). To have a limit at \( a \), a function must be defined everywhere in an open interval containing \( a \) (except possibly at \( a \) itself) and \( L \) must be a number.

We say,

\[
\lim_{{x \to a}} f(x) = L
\]

if and only if

\[
\lim_{{x \to a^-}} f(x) = \lim_{{x \to a^+}} f(x) = L
\]
Below is the graph of $f(x) = \frac{1}{x-4}$.

What is $\lim_{x \to 4^+} f(x)$? $\lim_{x \to 4^-} f(x)$? $\lim_{x \to 4} f(x)$?
More examples:

1. \( \lim_{x \to 5} 2x + 1 \)

2. \( \lim_{x \to 3} \frac{x^2 - x - 6}{x - 3} \)

3. \( \lim_{x \to 0} \sqrt{x - 6} \)

4. \( \lim_{x \to 6^+} \sqrt{x - 6} \)
So, what conditions would make a limit not exist?
Express this limit in words and interpret its meaning:

\[
\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = 4
\]
4. Evaluate the limit: \( \lim_{x \to -4} \left( \frac{5}{x + 4} \right) \)
5. Evaluate the limit: $\lim_{x \to 0} \left( \frac{6x^2 - 7x}{x} \right)$
7. Evaluate the limit: \( \lim_{x \to -4} f(x) \). Given that

\[
f(x) = \begin{cases} 
4x & x < -4 \\
-16 & x > -4 
\end{cases}
\]
10. Evaluate the limit: \( \lim_{{x \to 2^+}} f(x) \). Given that

\[
f(x) = \begin{cases} 
4x + 2 & x \leq 2 \\
(x^2 - x) & x > 2 
\end{cases}
\]
Section 1.3 - Definition of Limit and Arithmetic Rules

**Uniqueness of a limit**

If \( \lim_{x \to c} f(x) = L \) and \( \lim_{x \to c} f(x) = M \) then \( L = M \).

**The limit of a sum:**

If \( \lim_{x \to c} f(x) = L \) and \( \lim_{x \to c} g(x) = M \) then \( \lim_{x \to c} (f(x) + g(x)) = L + M \) (provided each limit exists).

**The limit of a difference:**

If \( \lim_{x \to c} f(x) = L \) and \( \lim_{x \to c} g(x) = M \) then \( \lim_{x \to c} (f(x) - g(x)) = L - M \) (provided each limit exists).
The limit of a product:

If \( \lim_{x \to c} f(x) = L \) and \( \lim_{x \to c} g(x) = M \) then \( \lim_{x \to c} (f(x)g(x)) = L \cdot M \)
(provided each limit exists).

The limit of a quotient:

If \( \lim_{x \to c} f(x) = L \) and \( \lim_{x \to c} g(x) = M \) with \( M \neq 0 \), then \( \lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{M} \)
(provided each limit exists).
Note:

If \( \lim_{x \to c} f(x) = L \) and \( \lim_{x \to c} g(x) = M \) with \( L \neq 0 \) and \( M = 0 \), then \( \lim_{x \to c} \frac{f(x)}{g(x)} \) does not exist.
Section 1.3 - Definition of Limit and Arithmetic Rules

Graph \( f(x) = 2 \)

What is \( \lim_{x \to 4} f(x) \)?
What is \( \lim_{x \to -1} f(x) \)?
What is \( \lim_{x \to 0} f(x) \)?

So, the limit of a constant function is that function:
If \( f(x) = k \) then \( \lim_{x \to a} k = k \)
Graph \( f(x) = x \)

What is \( \lim_{x \to 4} f(x) \)?

What is \( \lim_{x \to -1} f(x) \)?

What is \( \lim_{x \to 0} f(x) \)?

So: If \( f(x) = x \) then \( \lim_{x \to a} x = a \)
What is a polynomial function?

A polynomial function is any function of the form:

\[ f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \]

Where \( a_k \) is a real number and \( n \) is an integer.

Examples:

\[ f(x) = x^2 - 4 \]

\[ g(x) = \frac{1}{3} x^5 + 2x^3 - x + 1 \]

\[ h(x) = 3x \]
Because a polynomial function is just a combination of linear functions and constants and we know that we can find the limit for linear and constant functions, we can easily find the limit of polynomial functions.

So, to find \( \lim_{x \to a} P(x) \), where \( P(x) \) is a polynomial function, just plug in \( a \). In other words,

\[
\lim_{x \to a} P(x) = P(a)
\]
Section 1.3 - Definition of Limit and Arithmetic Rules

What is a rational function?

A rational function, \( R(x) \) is the quotient of two polynomial functions:

\[
R(x) = \frac{P(x)}{Q(x)}
\]

Recall, **The limit of a quotient:**

If \( \lim_{{x \to c}} f(x) = L \) and \( \lim_{{x \to c}} g(x) = M \) with \( M \neq 0 \), then \( \lim_{{x \to c}} \frac{f(x)}{g(x)} = \frac{L}{M} \)

So, providing \( \lim_{{x \to c}} Q(x) \neq 0 \), we can just plug in to find the answer for a limit of a rational function.

However, if \( \lim_{{x \to c}} Q(x) = 0 \), we need to consider other things.
Let $P(x)$ and $Q(x)$ be polynomial functions and let $a$ be a real number. Then,

\[
\lim_{x \to a} \frac{P(x)}{Q(x)} = \begin{cases} 
\frac{P(a)}{Q(a)} & \text{if } Q(a) \neq 0 \\
\text{undefined} & \text{if } P(a) \neq 0 \text{ and } Q(a) = 0
\end{cases}
\]

If $P(a)$ and $Q(a)$ both equal 0 then more work is required.
Section 1.3 - Definition of Limit and Arithmetic Rules

Techniques to evaluate limits:

- direct substitution
- cancellation
- rationalization
- algebraic simplification
Section 1.3 - Definition of Limit and Arithmetic Rules

Examples:

1. \( \lim_{x \to -2} (3x^2 + 1) = \)

2. \( \lim_{x \to 0} \frac{x^2 - 2x}{2x + 1} = \)

3. \( \lim_{x \to -1} \frac{2x^2 - x - 3}{x + 1} = \)