Office Hours: Tuesdays and Thursdays 12:30-2pm
(also available by appointment)
Office: 218C PGH
Course webpage: www.casa.uh.edu
Determine if the following function is continuous at the point where \( x = 3 \).

\[
g(x) = \begin{cases} 
2x^2 + 9 & x < 3 \\
27 & x = 3 \\
x^3 & x > 3 
\end{cases}
\]
Section 1.4 - Continuity

2 Discuss the continuity of \( f(x) = \begin{cases} 
-x^2 & x < -1 \\
3 & x = -1 \\
2 - x & -1 < x \leq 1 \\
\frac{1}{x^2} & x > 1 
\end{cases} \)
A very important result of continuity is the **Intermediate Value Theorem**.

If \( f(x) \) is continuous on the closed interval \([a, b]\) and \( K \) is a value between \( f(a) \) and \( f(b) \), then there is at least one value \( c \) in \((a, b)\) such that \( f(c) = K \).
Section 1.5 - The Intermediate Value Theorem

Examples:
Use the intermediate value theorem to show that there is a solution to the given equation in the indicated interval.

1. \(2 \tan(x) - x = 1\) on the interval \([0, \frac{\pi}{4}]\)

2. Show there is a value of \(x\) between 1 and 3 so that \(-3x^3 + 2x^4 = 7\)
Does the Intermediate Value Theorem guarantee a solution to
$0 = x^2 + 6x + 10$ on the interval $[-1, 3]$?
Does the Intermediate Value Theorem guarantee a solution to \( f(x) = 0 \) for \( f(x) = 2\sin(x) - 8\cos(x) - 3x^2 \) on the interval \([0, \frac{\pi}{2}]\)?
Verify that the IVT applies to this function on the indicated interval and find the value of $c$ guaranteed by the theorem.

$$f(x) = x^2 - 3x + 1$$ on the interval $[0, 6]$, $f(c) = 5$. 
Section 1.5 - The Intermediate Value Theorem

The Intermediate Value Theorem also helps us solve polynomial and rational inequalities.

Examples:

1. \((x + 2)^2(3x - 2)(x - 1)^3 \leq 0\)
Section 1.5 - The Intermediate Value Theorem

\[ \frac{2x - 8x^2}{(x + 1)^2} \geq 0 \]
Section 1.5 - The Intermediate Value Theorem

\[ \frac{1}{x - 1} + \frac{1}{x + 2} < 0 \]
Section 1.5 - The Intermediate Value Theorem

\[
\frac{4}{x+1} - \frac{3}{x+2} \geq 1
\]
Why did we just work these problems?

These inequalities are able to be solved because of the Intermediate Value Theorem (IVT). The IVT basically states that if \( f(x) \) is continuous from \( x = a \) to \( x = b \), then you must pass through all points \( x = \) "c" plotted along the graph of \( f(x) \).

Note: Functions with complex roots do not meet the requirements of the IVT. Why??
Section 1.5 - The Intermediate Value Theorem

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The **Extreme Value Theorem** states:

If a function $f(x)$ is continuous on a closed interval $[a, b]$, then $f(x)$ has both a maximum and a minimum on $[a, b]$. 
Examples:
State whether it is possible to have a function $f$ defined on the indicated interval and meets the given conditions:

1. $f$ is defined on $[3, 6]$, $f$ is continuous on $[3, 6]$, takes on the values $-3$ and $3$ but does not take on the value $0$. 
Section 1.5 - The Extreme Value Theorem

2. \( f \) is defined on \([4, 5]\), \( f \) is continuous on \([4, 5)\), has a minimum value of 4 when \( x = 5 \) and no maximum value.
Section 1.5 - The Extreme Value Theorem

\( f \) is defined on \([2, 7]\), \( f \) is continuous on \([2, 7]\), is non-constant and takes on only integer values.
Section 1.5 - The Extreme Value Theorem

4. $f$ is defined on $[-1, 3]$, $f$ is not continuous on $[-1, 3]$ and takes on only two distinct values.
Section 1.6 - The Pinching Theorem; Trig Limits

Suppose $f(x)$, $g(x)$ and $h(x)$ are defined on an open interval containing $x = c$ (except possibly at $x = c$).

If $f(x) \leq g(x) \leq h(x)$ and $\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L$, then $\lim_{x \to c} g(x) = L$. 

![Figure 2.5.1](image)
Section 1.6 - The Pinching Theorem; Trig Limits

Note: Trigonometric functions are continuous on their domain:

\[
\lim_{{x \to c}} \sin(x) = \sin(c) \quad \lim_{{x \to c}} \cos(x) = \cos(c)
\]

Also, recall:

\[
\sin(0) = 0 \text{ and } \cos(0) = 1
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Now, I will use the Pinching Theorem to show:

\[
\lim_{x \to 0} \frac{\sin(x)}{x} = 1
\]
We are going to look at a wedge of the unit circle:
Section 1.6 - The Pinching Theorem; Trig Limits

So,

\[ \text{area of } \triangle POA < \text{area of sector } POA < \text{area of } \triangle QOA \]
For any number $a \neq 0$, we have:

\[
\lim_{x \to 0} \frac{\sin(ax)}{ax} = 1 \quad \text{and} \quad \lim_{x \to 0} \frac{1 - \cos(ax)}{ax} = 0
\]
Section 1.6 - The Pinching Theorem; Trig Limits

Examples:

1. \[ \lim_{x \to 0} \frac{\sin(5x)}{5x} = \]

2. \[ \lim_{x \to 0} \frac{\sin(5x)}{x} = \]

3. \[ \lim_{x \to 0} \frac{\sin(5x)}{2x} = \]
Section 1.6 - The Pinching Theorem; Trig Limits

Examples:

4. \( \lim_{{x \to 0}} \frac{x}{\sin(x)} = \)

5. \( \lim_{{x \to \pi/4}} \frac{\sin(2x)}{x} = \)

6. \( \lim_{{x \to 0}} \frac{1 - \cos(3x)}{x} = \)
To Do

Read Sections 1.4-1.6.

Start hw 2

Work on quizzes 3, 4 and 5.

Register for Test 1.

It helps to do the work problems right after the lesson.

Post questions to the discussion board on CASA.