Math 1431
Section 16679

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Questions
Popper 01

Poppers start today!
Evaluate: \( \lim_{{x \to 9}} \frac{x - 3}{\sqrt{x} - 3} \)
Section 1.4 - Continuity

Continuity

A function $f$ is said to be continuous at a point $c$ if

1. $f(c)$ is defined.

2. $\lim_{x \to c} f(x)$ exists.

3. $\lim_{x \to c} f(x) = f(c)$. 
Section 1.4 - Continuity

Types of discontinuity at a point

1. Removable:

2. Non-Removable - Jump:
Section 1.4 - Continuity

Types of discontinuity at a point

1. Non-Removable - Infinite:
Section 1.4 - Continuity

\[ f(x) = \sqrt{x - 3} \]
Section 1.4 - Continuity

6. \( f(x) = \frac{\sqrt{x} - 1}{x^2 + 4x - 5} \)
Section 1.4 - Continuity

\[ g(x) = \begin{cases} 
  x + 2 & x < -2 \\
  \sqrt{4 - x^2} & -2 \leq x < 2 \\
  1 & x = 2 \\
  x - 2 & x > 2 
\end{cases} \]
Discuss the continuity of \( f(x) = \begin{cases} 
-x^2 & x < -1 \\
3 & x = -1 \\
2 - x & -1 < x \leq 1 \\
\frac{1}{x^2} & x > 1 
\end{cases} \)
Section 1.5 - The Intermediate Value Theorem

A very important result of continuity is the Intermediate Value Theorem.

If \( f(x) \) is continuous on the closed interval \([a, b]\) and \( K \) is a value between \( f(a) \) and \( f(b) \), then there is at least one value \( c \) in \((a, b)\) such that \( f(c) = K \).
Section 1.5 - The Intermediate Value Theorem

Examples:
Use the intermediate value theorem to show that there is a solution to the given equation in the indicated interval.

1. \( x^2 - 4x + 3 = 0 \) on the interval \([2, 4]\)

2. \( x^3 - 6x^2 - x + 2 = 0 \) on the interval \([0, 3]\)
3. $2 \tan(x) - x = 1$ on the interval $[0, \frac{\pi}{4}]$

4. Show there is a value of $x$ between 1 and 3 so that $-3x^3 + 2x^4 = 7$
Does the Intermediate Value Theorem guarantee a solution to 
\[0 = x^2 + 6x + 10\] on the interval \([-1, 3]\)?
Does the Intermediate Value Theorem guarantee a solution to $f(x) = 0$ for $f(x) = 2\sin(x) - 8\cos(x) - 3x^2$ on the interval $[0, \frac{\pi}{2}]$?
Verify that the IVT applies to this function on the indicated interval and find the value of $c$ guaranteed by the theorem.

$$f(x) = x^2 - 3x + 1$$ on the interval $[0, 6]$, $f(c) = 5$. 
The Intermediate Value Theorem also helps us solve polynomial and rational inequalities.

Examples:

\[ (x + 2)^2(3x - 2)(x - 1)^3 \leq 0 \]
\[ \frac{2x - 8x^2}{(x + 1)^2} \geq 0 \]
Section 1.5 - The Intermediate Value Theorem

\[ \frac{1}{x - 1} + \frac{1}{x + 2} < 0 \]
Section 1.5 - The Intermediate Value Theorem

\[
\frac{4}{x + 1} - \frac{3}{x + 2} \geq 1
\]
Section 1.5 - The Intermediate Value Theorem

Why did we just work these problems?

These inequalities are able to be solved because of the Intermediate Value Theorem (IVT). The IVT basically states that if \( f(x) \) is continuous from \( x = a \) to \( x = b \), then you must pass through all points \( (x = c) \) plotted along the graph of \( f(x) \).

Note: Functions with complex roots do not meet the requirements of the IVT. Why??
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Note: Functions with complex roots do not meet the requirements of the IVT. Why??
Does the IVT guarantee a solution to the equation
\[ f(x) = \frac{x^2 - 2x + 1}{x - 1} \]
on the interval \([0, 3]\).
The Extreme Value Theorem states:

If a function \( f(x) \) is continuous on a closed interval \([a, b]\), then \( f(x) \) has both a maximum and a minimum on \([a, b]\).
Evaluate the difference quotient \( \left( \frac{f(x + h) - f(x)}{h} \right) \) for 

\( f(x) = 2x + 3 \) at \( x = 1 \).
We will be measuring how \( f(x) \) changes when \( x \) changes but first, we need to understand slope a little more.

Slope of a secant line:

\[
\frac{f(B) - f(A)}{B - A}
\]

Also indicates the average rate of change over those values.
Section 2.1 - The Derivative

In calculus, we are more concerned with *instantaneous rate of change*, or rather the rate of change at a single point. To understand this, we will look at our secant line above and move \( A \) and \( B \) *very* close together - so close, the distance between \( A \) and \( B \) is near 0. Let’s let the distance between \( A \) and \( B \) be \( h \) so we have \( B = A + h \). Now our slope formula becomes:

\[
\frac{f(A + h) - f(A)}{(A + h) - A}
\]

or rather,

\[
\frac{f(A + h) - f(A)}{h}
\]
So, as this distance between $A$ and $B$ gets close to 0, we can say $h \to 0$. 
Section 2.1 - The Derivative

So, as this distance between $A$ and $B$ gets close to 0, we can say $h \to 0$. Our graph will now look like this:
Section 2.1 - The Derivative

The slope of this tangent line at the point $x = A$ is the instantaneous rate of change at $x = A$. This is denoted with $f'(A)$ and is found by this formula:

$$f'(A) = \lim_{h \to 0} \frac{f(A + h) - f(A)}{h}$$
Section 2.1 - The Derivative

Examples:

1. Find the slope of the tangent line at $x = 3$ for $f(x) = x^2 + 1$
Find the slope of the tangent line at $x = 1$ for $f(x) = \sqrt{x}$
The value of the derivative at $x = 2$ for $f(x) = \frac{1}{x + 1}$ is the ___ of the tangent line at $x = 2$. 
The Definition of Derivative

A function $f(x)$ is differentiable at $x$ if and only if

$$
\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
$$

exists. In this case, we denote

$$
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
$$

and we refer to $f'(x)$ as the derivative of $f$ at $x$. $f'(x)$ can be thought of as the slope function. It gives the slope of the graph of $f(x)$ at any point $x$.

The derivative can also be denoted as:

$$
f'(x), \quad y', \quad \frac{dy}{dx}, \quad \frac{d}{dx}[f(x)]
$$
Find the derivative of $f(x) = x^2 - 2x$ using the definition of the derivative.
Find the slope of the tangent line to $f(x) = x^2 - 2x$ at $x = 3$. 

(a) 1 
(b) 0 
(c) 6 
(d) 4 
(e) None of these