Office Hours: Tuesdays & Thursdays 11:45-12:45
(also available by appointment)
Office: 218C PGH
Course webpage: www.casa.uh.edu
Questions
The Power Rule:

\[ \frac{d}{dx} (x^n) = nx^{n-1}, \quad n \neq 0 \]
1 Find the derivative of \( f(x) = \frac{1}{x} + \sqrt{x} + x \).
Higher Order Derivatives:

\[ f'(x), \quad f''(x), \quad f'''(x), \quad f^{(4)}(x) \]

\[ \frac{d}{dx} f(x), \quad \frac{d^2}{dx^2} f(x), \quad \frac{d^3}{dx^3} f(x), \quad \frac{d^4}{dx^4} f(x) \]
Section 2.2 - Differentiation Formulas

Examples:

1. \[ \frac{d^2}{dx^2} (3x^3 - 5x^2 + 2x - 1) = \]

2. \[ \frac{d^3}{dx^3} (x^8 + 2x^5 - 2x + 5) = \]
Find \[ \frac{d^2}{dx^2} \left( \frac{2}{x} - x^5 \right) \]
Section 2.2 - Trig Derivatives

\[
\frac{d}{dx} \sin(x) = \cos(x)
\]
\[
\frac{d}{dx} \cos(x) = -\sin(x)
\]
\[
\frac{d}{dx} \tan(x) = \sec^2(x)
\]
\[
\frac{d}{dx} \cot(x) = -\csc^2(x)
\]
\[
\frac{d}{dx} \sec(x) = \sec(x) \cdot \tan(x)
\]
\[
\frac{d}{dx} \csc(x) = -\csc(x) \cdot \cot(x)
\]
Given $f(x) = x^2 + \cos(x)$, find $f'(x)$.
The Product Rule:

If \( f \) and \( g \) are differentiable, then \( f \cdot g \) is differentiable and

\[
\frac{d}{dx}(f(x) \cdot g(x)) = f(x) \cdot g'(x) + f'(x) \cdot g(x)
\]
Another way to write the Product Rule:

Suppose that \( f(x) = u \cdot v \) where \( u \) and \( v \) are differentiable functions of \( x \). Then,

\[
f'(x) = u' \cdot v + u \cdot v'
\]

or

\[
f'(x) = u \cdot v' + u' \cdot v
\]
Section 2.3 - Differentiation Rules

Proof: Let \( F(x) = f(x) \cdot g(x) \), then \( F(x + h) = f(x + h) \cdot g(x + h) \) and

\[
F'(x) = \lim_{h \to 0} \frac{f(x + h) \cdot g(x + h) - f(x) \cdot g(x)}{h} \\
= \lim_{h \to 0} \frac{f(x + h) \cdot g(x + h) - f(x + h) \cdot g(x) + f(x + h) \cdot g(x) - f(x) \cdot g(x)}{h} \\
= \lim_{h \to 0} \frac{f(x + h)[g(x + h) - g(x)] + [f(x + h) - f(x)]g(x)}{h} \\
= \left[ \lim_{h \to 0} f(x + h) \right] \cdot \left[ \lim_{h \to 0} \frac{g(x + h) - g(x)}{h} \right] \\
+ \left[ \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \right] \cdot \left[ \lim_{h \to 0} g(x) \right] \\
= f(x) \cdot g'(x) + f'(x) \cdot g(x)
\]
Examples: Find the derivative of each:

1. \( y = (5x + 2)(x^2 + 1) \)

2. \( f(x) = (3x - 1)(2x^4 - x) \)
Section 2.3 - Differentiation Rules

3. \( y = x^2 \cos(x) \)

4. \( f(x) = (x^2 - 2x + 1) \tan(x) \)
Find the derivative of $y = x^3 \cdot f(x)$.
Section 2.3 - Differentiation Rules

The Quotient Rule:

If \( f \) and \( g \) are differentiable, then \( \frac{f}{g} \) is differentiable (providing \( g \neq 0 \)) and

\[
\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}
\]
Another way to write the Quotient Rule:

Suppose that \( f(x) = \frac{u}{v} \) where \( u \) and \( v \) are differentiable functions of \( x \) and \( v \neq 0 \). Then,

\[
f'(x) = \frac{u' \cdot v - u \cdot v'}{v^2}
\]
Section 2.3 - Differentiation Rules

Examples: Find the derivative of each:

1. \( y = \frac{x}{x^2 + 1} \)

2. \( f(x) = \frac{x^2 - 4}{x - 3} \)
Section 2.3 - Differentiation Rules

3. \[ y = \frac{1}{x + 1} \]

4. \[ \frac{d}{dx} \left( \frac{x}{\sin(x)} \right) = \]
5) Consider the function \( f(x) = x^3 - 3x^2 + 3 \). Find the points where the tangent line is horizontal.
6) Given the function \( f(x) = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x + 2 \), find the points where the tangent line has slope \(-2\).
9) Determine the number(s), $x$, between 0 and $2\pi$ where the line tangent to the function $f(x) = 4\sqrt{3}\sin(x) + 4\cos(x)$ is horizontal.
Motivation behind the chain rule:
Find the derivative of each:

1. \( y = 5x^4 \)

2. \( y = (2x + 1)^2 \)

3. \( y = (2x + 1)^{14} \)
Recall:

Composite functions are functions within functions. They are written $f(g(x))$ or $(f \circ g)(x)$.

For example:
If $f(x) = 3x - 4$ and $g(x) = x^2$

then $f(g(x)) = 

and $g(f(x)) = 

To find the derivative of composite functions, we use the chain rule.
The Chain Rule:
Let \( f(x) \) and \( g(x) \) be separate functions of \( x \) and let \( y = f(g(x)) \), then

\[
y' = f'(g(x)) \cdot g'(x)
\]
or

\[
\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}
\]

Where in \( \frac{dy}{du} \) we have substituted \( y = f(x) \) and \( u = g(x) \).
Section 2.3 - Differentiation Rules

Examples: Find the derivative of each:

1. \( y = (2x + 1)^{14} \)

2. \( f(x) = (x - 1)^5 \)
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3. \( y = (x^2 + 6x - 4)^3 \)

4. \( g(x) = \sqrt{x^2 + 3} \)
Section 2.3 - Differentiation Rules

5 \[ y = 6(3x + 2)^4 \]

6 \[ g(x) = \sin^2(x) \]
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\[ f(x) = \left( x^2 + \frac{1}{x^2} \right)^3 \]
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\[ f(x) = \left( \frac{x}{2x^2 + 1} \right)^3 \]
9 \ y = \cos (\sqrt{x})
Suppose \( G(x) = f(h(x)) \) with \( h(1) = 2, \ f'(1) = 3, \ f'(2) = -6, \ h'(1) = 7. \) Find \( G'(1). \)
Find $\frac{d}{dx} (f(g(x)))$. 