Office Hours: Tuesdays and Thursdays 12:30-2pm
(also available by appointment)
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Questions
At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$?
Section 3.3 - Increasing and Decreasing Functions

In plain terms, a function is increasing if, as $x$ moves to the right, its graph moves up, and is decreasing if its graph moves down.

A function is strictly monotonic on an interval if it is either increasing on the entire interval or decreasing on the entire interval.

Since $f'(a)$ gives us the slope of the tangent line to $f(x)$ at $x = a$, it follows that:

where $f'(x)$ is positive, $f(x)$ is increasing

and

where $f'(x)$ is negative, $f(x)$ is decreasing.
Section 3.3 - Increasing and Decreasing Functions

In math terms

\[ f \text{ is increasing over an interval } I \text{ if and only if } f(a) < f(b) \text{ for all } a, b \in I \text{ with } a < b. \]

Theorem: A function \( f \) is increasing on an interval \( I \) provided \( f \) is continuous and \( f'(x) > 0 \) at all but finitely many values in \( I \).
Section 3.3 - Increasing and Decreasing Functions

In math terms

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if and only if

\[ f(a) < f(b) \]

for all \( a, b \in I \) with \( a < b \).
Section 3.3 - Increasing and Decreasing Functions

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Theorem: A function \( f \) is increasing on an interval \( I \) provided \( f \) is continuous and \( f'(x) > 0 \) at all but finitely many values in \( I \).
And...

\[ f \text{ is decreasing over an interval } I \]
\[ \text{if and only if} \]
\[ f(a) > f(b) \]
\[ \text{for all } a, b \in I \text{ with } a < b. \]
Section 3.3 - Increasing and Decreasing Functions

And...

\[ f \text{ is decreasing over an interval } I \]
\[ \text{if and only if} \]
\[ f(a) > f(b) \]
\[ \text{for all } a, b \in I \text{ with } a < b. \]

Theorem: A function \( f \) is increasing on an interval \( I \) provided \( f \) is continuous and \( f'(x) < 0 \) at all but finitely many values in \( I \).
Definition of Critical Number:

The numbers \( c \) in the domain of a function \( f \) for which either \( f'(c) = 0 \) or \( f'(c) \) does not exist, are called the critical numbers of \( f \).

The terms critical points and critical values are also used.
Examples:

1. Find the critical numbers of $f(x) = 3x^4 - 4x^3$. 
Find the critical numbers of \( f(x) = \frac{x - 1}{x - 3} \).
Find the critical numbers of \( f(x) = (x^2 - 36)^{1/3} \).
Find all critical numbers: \( f(x) = \frac{x^2}{x^2 - 4} \)
Section 3.3 - Increasing and Decreasing Functions

Theorem: Test for Increasing or Decreasing Functions

Let \( f \) be a function that is continuous on the closed interval \([a, b]\) and differentiable on the open interval \((a, b)\).

1. If \( f'(x) > 0 \) for all \( x \) in \((a, b)\), then \( f \) is increasing on \([a, b]\).
2. If \( f'(x) < 0 \) for all \( x \) in \((a, b)\), then \( f \) is decreasing on \([a, b]\).
3. If \( f'(x) = 0 \) for all \( x \) in \((a, b)\), then \( f \) is constant on \([a, b]\).

To find intervals on which a continuous function is increasing or decreasing:

1. Locate the critical numbers to determine test intervals.
2. Determine the sign of \( f'(x) \) at one value in each interval.
3. Using the previous theorem, determine if the function is increasing or decreasing on the interval.
Section 3.3 - Increasing and Decreasing Functions

Examples: Determine the intervals of increase and/or decrease for each of the following.

\[ f(x) = \frac{1}{3} x^3 + \frac{5}{2} x^2 + 6x - 3. \]
Section 3.3 - Increasing and Decreasing Functions

$f(x) = \frac{x^2 + 1}{x^2 - 1}$
Section 3.3 - Increasing and Decreasing Functions

3. \[ f(x) = \frac{2x}{x^2 - 4} \]
Where is $f(x)$ decreasing?

(a) $[-2, 1] \cup [1, 2]$

(b) $x = \pm 1$

(c) $[1, 1]$

(d) None of these.
Find all critical numbers for $f(x) = (9 - x^2)^{3/5}$
If this is the graph of $f'(x)$, how many critical numbers are there?
Where are the local “extremes” of the function shown? What can be said about the values of the derivatives at those points?
Section 3.4 - Extreme Values

The graph of $f'(x)$ is shown. Give a possible sketch for $f(x)$. 

![Graph of $f'(x)$]
Section 3.4 - Extreme Values

How can we classify critical points as either local (or relative) maximums or local minimums?

First Derivative Test:

Let $c$ be a critical number of a function $f$ that is continuous on an open interval $I$ containing $c$. If $f$ is differentiable on the interval, except possibly at $c$, then $f(c)$ can be classified as follows:

1. If $f'(x)$ changes from negative to positive at $c$, then $f(c)$ is a local minimum of $f$.
2. If $f'(x)$ changes from positive to negative at $c$, then $f(c)$ is a local maximum of $f$.

When using the First Derivative Test, be sure to consider the domain of the function. The $x$ value where the function is undefined must be used with the critical numbers to determine the test intervals.
Section 3.4 - Extreme Values

How can we classify critical points as either local (or relative) maximums or local minimums?

First Derivative Test:

Let \( c \) be a critical number of a function \( f \) that is continuous on an open interval \( I \) containing \( c \). If \( f \) is differentiable on the interval, except possibly at \( c \), then \( f(c) \) can be classified as follows:

1. If \( f'(x) \) changes from negative to positive at \( c \), then \( f(c) \) is a local minimum of \( f \).

2. If \( f'(x) \) changes from positive to negative at \( c \), then \( f(c) \) is a local maximum of \( f \).

When using the First Derivative Test, be sure to consider the domain of the function. The \( x \)-value where the function is undefined must be used with the critical numbers to determine the test intervals.
A slope chart is shown below for the function $f$. Classify the critical point at $x = 0$.

$$
\begin{array}{c|c|c}
\text{x} & 0 & 2 \\
\hline
f''(x) & \--- & ++++++ & \--- \\
\end{array}
$$
A slope chart is shown below for the function $f$. Classify the critical point at $x = 2$.

- For $x < 0$, $f''(x) < 0$.
- For $0 < x < 2$, $f''(x) > 0$.
- For $x > 2$, $f''(x) < 0$.

This indicates that $x = 2$ is a local minimum.
Examples: For each function below, find the critical numbers, the intervals on which the function is increasing or decreasing. Locate all local extrema.

1. \( f(x) = \frac{1}{2}x^3 - 6x \)
Section 3.4 - Extreme Values

2. \( f(x) = \frac{x^4 + 1}{x^2} \)
Section 3.4 - Extreme Values

If $f' > 0$, what conclusion can be made about $f$?

If $f' < 0$, what conclusion can be made about $f$?

If $f'' > 0$, what conclusion can be made about $f'$?

If $f'' < 0$, what conclusion can be made about $f'$?
So, if $f'(c) = 0$ and $f''(c) > 0$, what conclusion can be made about $f(c)$?

And, if $f'(c) = 0$ and $f''(c) < 0$, what conclusion can be made about $f(c)$?
Section 3.4 - Extreme Values

Thm: The Second Derivative Test

Let $f$ be a function such that $f'(c) = 0$ and the second derivative of $f$ exists on an open interval containing $c$.

1. If $f''(c) > 0$, then $f(c)$ is a local minimum.
2. If $f''(c) < 0$, then $f(c)$ is a local maximum.
3. If $f''(c) = 0$, then the test fails. In such cases, you can use the First Derivative Test.
How to use the Second Derivative Test:

- Determine the critical numbers using the first derivative.
- Plug these numbers into the second derivative and get the value.
- If the value is positive, you have a relative minimum.
- If the value is negative, you have a relative maximum.
- If the value is zero, use the First Derivative Test to determine if there is a local max or min.
Examples:

1. Locate the local extrema for \( f(x) = 2x^3 + 3x^2 - 12x \)
2 Locate the local extrema for \( f(x) = 2\sin(x) + \cos(2x) \) on \((0, 2\pi)\)
Find the critical numbers, the intervals on which the function is increasing or decreasing, and all local extrema.

\[ f(x) = -(x - 1)^2(x + 2) \]
Locate the local extrema for \( f(x) = \frac{x^2}{x^2 - 9} \)
Suppose $x = 3$ is a critical number for $f(x)$ and that $f''(x) = x^2 - 2x + 2$, classify this critical number.
Let \( f(x) \) be a polynomial function with \( x = 1 \) as a critical number. If \( f''(1) > 0 \), then which of the following statements is true?
Which of the following functions fails to satisfy the conditions of The Mean Value Theorem on the given interval?

(a) \( f(x) = \frac{3x^2}{3} \) on \([1, 3]\)

(b) \( f(x) = |3x^2| \) on \([-2, 3]\)

(c) \( f(x) = 5x^4 - 5x^2 \) on \([5, 0]\)

(d) \( f(x) = \sqrt{x^2} \) on \([1, 3]\)

(e) More than one of these.
To Do

Read section 3.1-3.3.

Schedule Test 2.

Work quiz 10 and 11.

Do homework and send me questions.