Math 1431
Section 16679

Bekki George: rageorge@central.uh.edu

University of Houston

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Office Hours: Tuesdays & Thursdays 11:45-12:45 (Not 10/15) (also available by appointment)
Office: 218C PGH
Course webpage: www.casa.uh.edu
True or false? A circle is a function. (example: $x^2 + y^2 = 4$)
What determines whether the graph of a function is invertible (has an inverse that is also a function)?

**Definition:** A function $f$ is one-to-one if $f(x_1) = f(x_2)$ then $x_1 = x_2$. In other words, two different $x$ values cannot have the same $y$ values.

If a function is one-to-one, then it has an inverse. (Remember, domain of $f$ equals the range of $f^{-1}$)
Section 4.1 - Inverses

Which of the following functions are invertible?
Section 4.1 - Inverses

Theorem: If $f$ is either an increasing function or a decreasing function, then $f$ is an invertible function.
Example: Show that $f(x) = x^3 + 3x$ is invertible on the interval $[0, 10]$. 
2 True or false? A parabola has an inverse. (example: $y = 4x^2$)
Section 4.1 - Inverses

Theorem: If $f$ is either an increasing function or a decreasing function, then $f$ is an invertible function.
Example: Show that $f(x) = x^3 + 3x$ is invertible on the interval $[0, 10]$. 
Section 4.1 - Inverses

Example: Show that \( f(x) = \sin(x) \) is invertible on the interval \( [-\frac{\pi}{2}, \frac{\pi}{2}] \).
Section 4.1 - Inverses

How do we find the formula for the inverse of a function?

1. Start with \( y = f(x) \).
2. Solve for \( x \) in terms of \( y \). This will give something like \( x = g(y) \).
3. Switch the \( x \)'s and \( y \)'s. This will give \( y = g(x) \).
4. The function \( g \) is the inverse of \( f \).

We can only do this for simple functions.

We will use the notation \( f^{-1}(x) \) to denote the inverse of \( f(x) \).
Section 4.1 - Inverses

Example: Is $f(x) = 2x - 3$ invertible? If so, find its inverse.
Example: Find the inverse of \( y = \frac{x + 2}{x - 3} \) if possible.
Section 4.1 - Inverses

How are functions related to their inverses?

Algebraically:

Geometrically:
Section 4.1 - Inverses

**Theorem:** If $f(x)$ is continuous and invertible then $f^{-1}(x)$ is continuous.

**Theorem:** If $f(x)$ is differentiable and invertible, and $f'(x)$ is nonzero, then $f^{-1}(x)$ is differentiable.

Also, if $f(a) = b$ and $f'(a) \neq 0$, then $(f^{-1})'(b) = \frac{1}{f'(f^{-1}(b))} = \frac{1}{f'(a)}$. 
Section 4.1 - Inverses

We found that \( f(x) = x^3 + 3x \) was invertible on \([0, 10]\). Find \((f^{-1})'(4)\).
Section 4.1 - Inverses

Given \( f(x) = 2x + \sin(x) \), find \( (f^{-1})' (2\pi) \) (if possible).
Section 4.1 - Inverses

Let \( f(x) = x^5 + 2x^3 + 2x \). Give an equation of the tangent line to the graph of \( f^{-1}(x) \) at the point \((-5, -1)\).
Section 4.1 - Inverses

Given $f(x) = x^5 + 1$, find $(f^{-1})'(33)$ if possible.
Is \( f(x) = x^3 + 2x - 3 \) invertible?
Find \((f^{-1})'(2)\) if
\[ f(2) = 3, \quad f(4) = 2, \quad f(3) = -2, \quad f'(2) = 7, \quad f'(3) = 5, \quad f'(4) = 10. \]
To Do

Read 4.1.

Take quiz 15.

Email questions if you have any.