Math 1432

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Class webpage:
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More IBP examples:

\[
\int u \, dv = uv - \int v \, du
\]

1. \[\int_{0}^{\pi/2} x^2 \sin x \, dx\]
2. \( \int (e^x + 2x)^2 \, dx \)
3. \( \int x^2 \arctan x \, dx \)
1. \[ \int xe^x \, dx = \]

2. Rewrite \( \cos^2 x \) in terms of sine.
8.2 Powers and Products of Trigonometric Functions

Recall the following identities:

\[ \cos^2(x) + \sin^2(x) = 1 \]
\[ 1 + \tan^2(x) = \sec^2(x) \]
\[ 1 + \cot^2(x) = \csc^2(x) \]

\[ \cos^2(x) = \frac{1 + \cos(2x)}{2} \quad \sin^2(x) = \frac{1 - \cos(2x)}{2} \]

\[ \sin(2x) = 2 \sin x \cos x \]
\[ \cos(2x) = \cos^2 x - \sin^2 x \]
In this section, we will study techniques for evaluating integrals of the form

\[ \int \sin^m x \cos^n x \, dx \quad \text{and} \quad \int \sec^m x \tan^n x \, dx \]

where either \( m \) or \( n \) is a positive integer.

To find antiderivatives for these forms, try to break them into combinations of trigonometric integrals to which you can apply the Power Rule, which is

\[
\int u^n \, du = \begin{cases} 
\frac{u^{n+1}}{n+1} + C & \text{if } n \neq 1 \\
\ln|u| + C & \text{if } n = -1.
\end{cases}
\]
Integrals Involving Powers of Sine and Cosine

1. If \( m \) or \( n \) odd:
   a. \( m \) odd: rewrite \( \sin^m x \) as \( \sin^{m-1} x \sin x \) \((m-1) \text{ is even so can use identity} \)
      \( \sin^2 x = 1 - \cos^2 x \)
   b. \( n \) odd: rewrite \( \cos^m x \) as \( \cos^{m-1} x \cos x \) \((n-1) \text{ is even so can use identity} \)
      \( \cos^2 x = 1 - \sin^2 x \)

Examples:

\[
\int \sin^3 x \, dx
\]
\[ \int \sin^3 x \cos^2 x \, dx \]
\[ \int \cos^5 x \, dx \]
\[ \int \sin^4 x \cos^5 x \, dx \]
2. If $m$ and $n$ even use these identities:

\[
\sin^2 x = \frac{1 - \cos 2x}{2} \quad \text{and} \quad \cos^2 x = \frac{1 + \cos 2x}{2}.
\]

\[
\int \cos^2 x \, dx
\]
Note:
\[
\int \sin^2 x \, dx = \frac{1}{2} x - \frac{1}{2} \sin x \cos x + C
\]
\[
\int \cos^2 x \, dx = \frac{1}{2} x + \frac{1}{2} \sin x \cos x + C
\]
Integrals involving Secants and Tangents

\[ \tan^2 x + 1 = \sec^2 x \]

For \( \int \tan^m x \sec^n x \, dx \)

a. \( n \) even: rewrite \( \tan^m x \sec^n x \) as \( \tan^m x \sec^{n-2} x \sec^2 x \) (then you can use identity \( \sec^2 x = \tan^2 x + 1 \))

b. \( m \) odd: rewrite as \( \tan^{m-1} x \sec^{n-1} x \cdot \sec x \tan x \) (\( m-1 \) is even so can use identity \( \tan^2 x = \sec^2 x - 1 \))

c. \( m \) even and \( n \) odd: rewrite \( \tan^m x \) using \( \tan^2 x = \sec^2 x - 1 \)
Examples:
\[ \int \tan^3(x) \, dx \]
\[ \int \sec^4 x \, dx \]
\[ \int \sec^4 x \tan^2 x \, dx \]
\[ \int \frac{\sec x}{\tan^2 x} \, dx \]
\int \tan^3 x \, dx
\[ \int \sec^5 x \tan x \, dx \]
Note:

\[ \int \tan^n x \, dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x \, dx \quad n \geq 2 \]

\[ \int \sec^n x \, dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx \quad n \geq 2 \]
3. \[ \int \cos x \sin^3 x \, dx \]

4. Compute \[ \int \sec(2x) \tan^3(2x) \, dx \]